ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ ТА МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ INFORMATION TECHNOLOGY AND MATHEMATICAL MODELING

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FINANCIAL RISK ANALYSIS IN MULTIDIMENSIONAL SYSTEMS

Abstract. *A new approach is proposed for modeling the interdependence among factors of multivariate risks, represented as matrices of interdependence measures for numerical description and a family of copulas with parameter estimates for analytical description. The approach proposed to construct a multivariate risk model in which, marginal distributions are modeled separately using elliptical distributions for measurements at the center of the samples and extreme distributions in the tails, while the dependencies between risks are modeled by copulas. The joint distribution is modeled using marginal distributions and copulas and can be applied to the analysis of risk characteristics. An approach to determining risk dependencies using the concept of mutual information within the framework of Bayesian networks has been developed. A computational experiment involving two generated, theoretically well-known three-dimensional distributions and one empirical three-dimensional distribution for exchange rates demonstrated the applicability of the proposed approach to modeling multidimensional risk.*

The problem of identifying the optimal portfolio structure under active risk management and asset liquidity constraints, a multidimensional model for estimating tail risk measures is proposed. A computational experiment conducted to estimate risk measures by generating a sample yielded an estimation error of less than one percent for non-extreme quantiles. The quality of the estimation of risk deviation measures requires further refinement of the model. The quality of risk measure estimates for the tail regions of distributions indicates that the model based on a combination of marginal distributions using normal and Pareto distributions needs to be improved to describe central observations.

Keywords: *system analysis, financial risks, multidimensional distribution, copula families, joint distribution, dependency measures, combined marginal distribution.*

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Introduction

Financial risk management remains an important problem for modern economic and financial systems, especially under conditions of non-stationary and nonlinear processes [1–3]. Effective risk analysis requires not only estimation of separate risks but also consideration of dependencies between multiple risk factors. Interaction between factors in high-dimensional systems may significantly increase total losses and complicate the process of risk estimation and decision-making [4, 5].

The behavior of financial positions depends on numerous exogenous and endogenous factors related to market conditions, economic sectors, and internal market dynamics [6]. As the number of factors increases, the probability of extreme events and heavy-tailed distributions also grows. Traditional low-dimensional models based on several principal factors remain popular due to their simplicity; however, they are often insufficient for describing complex dependency structures in large-scale systems [7].

To analyze dependencies between financial instruments, this study applies approaches based on random matrix theory and dependency measures. Previous studies demonstrated that empirical correlation matrices contain dominant eigenvalues that differ from the theoretical spectra of random matrices, indicating the existence of meaningful latent market factors [9–13]. Such analysis makes it possible to separate informative dependencies from noise and improve multivariate risk modeling.

Because the traditional linear correlation coefficient has limitations in describing nonlinear and tail dependencies, this work considers alternative dependency measures and copula-based approaches for modeling multivariate financial risks. The proposed methodology combines dependency analysis, multivariate distributions, and numerical simulation methods to support practical risk estimation and portfolio analysis in multidimensional financial systems.

Problem statement

The study is aiming to solving the following problems:

- to formulate systemic approach to analysis of risks for multivariate portfolio;
- determine appropriate dependency measures and the limits of their application to estimation the risk factors in numerical form;
- application of random matrices theory to studying the properties of dependency measures;

- to analyze the possibility of application the distributions of correlation matrices eigenvalues for different dependency measures and distributions of distances between the eigenvalues aiming to determine the number of principal factors in a model;
- to formulate the method for estimation risk characteristics using combined multivariate model and illustrate practical application of the method.

Material

Dependency measures. First, consider the very notion of dependency for random variables. Independent are random variables, $X_1, \ldots, X_n$, for which the following equality is true:

$$P(X_1 \leq x_1; \ldots; X_n \leq x_n) = P(X_1 \leq x_1) \cdot \ldots \cdot P(X_n \leq x_n)\ . \tag{1}$$

It means that the knowledge regarding one of the variables does not provide new knowledge about others. The dependency is inverse characteristic but the possibilities of its formulation can differ dependently on the problem statement.

The dependency measure should be symmetric and universal, i.e. applicable to any pair of continuous or discrete random variables, X and Y that characterize risks [15]. If dependence between risk factors does not exist the dependency measure should be equal zero, it should be restricted by the values [-1; 1], and reach minimum and maximum values when random variables are respectively, oppositely monotonic and congruent monotonic. Linear correlation completely characterizes dependency between normally distributed random values that model central part of combined distribution of risk. That is why any dependency measure should be expressed through Pearson coefficient of linear correlation in a case of two-dimensional normal distribution.

The dependency measure used in the models of financial and economic risks should be invariant with respect to continuous strictly increasing transforms and indicate to availability of differences between random variables as well as to be a measure of distance. If the distance measure has all mentioned properties it is called metrics of dependency.

Linear correlation. Linear correlation is most widely used dependency measure that is used in modeling multivariate economic and financial risks with the use of elliptical distributions, for example, normal and Student t-distribution. The coefficient of linear correlation between two random variables, X and Y, having finite standard deviations is computed via the expression:

$$\rho(X, Y) = \frac{E[XY] - E[X]E[Y]}{\sqrt{\sigma^2[X\sigma^2[Y]]}}\ , \tag{2}$$

where, $\sigma[X]$ and $\sigma[Y]$ are standard deviation of X and Y, respectively.

The coefficient of linear correlation is not metric of dependency. Because of usage of heavy tail distributions (and respectively infinite standard deviation) for which the measure (1) has not been determined, the linear correlation coefficient is not determined for all types of random values. This coefficient is invariant to strictly increasing linear transforms but in general case it is not invariant to nonlinear strictly increasing transforms. The coefficient of linear correlation is commutative, $\rho(X,Y)=\rho(Y,X)$ and restricted, $-1 \le \rho(X,Y) \le 1$; where equality is reached in a case of complete linear dependency of random values. Completely dependent random values can show correlation coefficient distinct from 1 or, -1. For independent random values the equality is true: $\rho(X,Y)=0$, however, from $\rho(X,Y)=0$does not follows independence of linked to them risks (for example, in the case of normally distributed X and completely depending from it risk X^2). The correlation coefficient does not provide for description of dependency between risks, especially in tails of a distribution (Fig. 1).

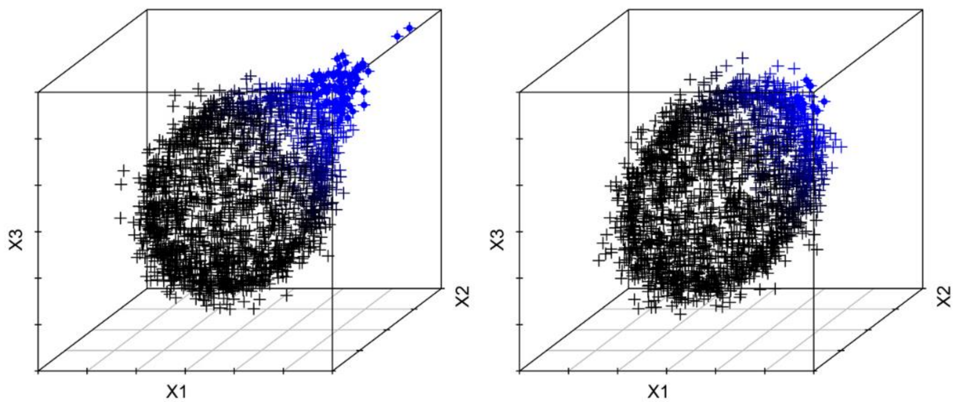

Fig. 1. Multivariate observations with similar normal marginal distributions and the same correlation coefficients, $\rho = 0.14$, but with different dependency structures

Concordance measures. Concordance of risks and corresponding variables means a tendency to simultaneous increasing or decreasing their values. The observations, (x_i, y_i) and (x_j, y_j) of the random values vector, (X,Y), are considered to be in concordance if, $x_i < x_j$, $y_i < y_j$ or $x_i > x_j$, $y_i > y_j$. The conditions of concordance for observations can be also written as follows: $(x_i - x_j)(y_i - y_j) > 0$; respectively, the condition of non-concordance is as follows: $(x_i - x_j)(y_i - y_j) < 0$. The coefficient of Kendall rank correlation, τ, and coefficient of rank Spearman correlation, ρ_S, are numerical measures of dependence that are linked to concordance measures. The concordance measure, τ, for samples of two random variables, X and Y, is determined as a difference between the number of pairs of two-dimensional observations that in concordance and the number of pairs that are not in concordance, divided by the total number of two-dimensional observation pairs.

If (X',Y') and (X'',Y'') are independent random vectors with similar distribution functions, then concordance measure Kendall, τ, is computed via the expression:

$$\tau = P[(X' - X'')(Y' - Y'') > 0] - P[(X' - X'')(Y' - Y'') < 0] . \qquad (3)$$

Under increasing transforms of ψ, ϕ and with $X' \ge X''$the following inequality is true: $\psi(X') \ge \psi(X'')$; and in analogy, with $Y' \ge Y''$ the following condition is true: $\phi(Y') \ge \phi(Y'')$. Thus, the Kendall, τ, is invariant to increasing transforms.

For two-dimensional normal distribution and for any random variables having dependency structure described by elliptical copula, the Kendall τ has an expression linking it with coefficient of linear correlation, ρ, as follows:

$$\tau = \frac{2}{\pi} \arcsin(\rho) . \quad (4)$$

The Spearman rank correlation, ρ_S, is also supported by the notion of concordance and non-concordance. However, this measure also takes into consideration marginal distributions of random variables. Let, (X', Y'), (X'', Y'') and (X''', Y'''), are independent random vectors with similar functions of joint distributions; then the Spearman concordance measure, ρ_S, is defined as follows:

$$\rho_S = P[(X' - X'')(Y' - Y'') > 0] - P[(X' - X'')(Y' - Y'') < 0] . \quad (5)$$

In the same way this measure of dependency can be determined using another component of the third vector, X'''. The coefficient of rank Spearman correlation is linked to the coefficient of linear Pearson correlation, ρ , as follows:

$$\rho_S = \frac{E(FG) - 1/4}{1/12} = \frac{E[FG] - E[F]E[G]}{\sqrt{\sigma^2[F]\sigma^2[G]}} = \rho(F, G), \quad (6)$$

where, F and G are marginal distribution functions of random variables X and Y, respectively.

The rank correlation coefficients, τ and, ρ_S, are commutative: $\tau(X,Y) = \tau(Y,X)$, $\rho_S(X,Y) = \rho_S(Y,X)$. For independent random variables, $\tau(X,Y) = \rho_S(X,Y) = 0$. The values of the both coefficients of rank correlation belong to the range: [-1,1]. These two concordance measures can be expressed via copula. This approach provides the possibility for using as numerical measure of dependence the concordance measures, Kendall, τ , and Spearman, ρ_S, in cases where traditionally is used linear correlation coefficient. In these cases it is also possible to remove the restrictions regarding normal or elliptical joint distribution.

Matrices of correlation coefficients. Consider the problem of determining the form of numerical description of dependency between more than two random variables. The dependency measure characterizes dependency structure between two random variables using one number. To model the risk of an organization the dependency measure is generalized for the case of, $N > 2$ (risks), i. e. the matrix, $N \times N$, of pairwise dependency measures is considered. Here empirical matrix of linear correlations is a key part of a model for computing the Value-at-Risk (VaR) measure for risks with normal distributions. According to Markowitz portfolio theory optimal portfolio corresponds to small eigenvalues of correlation matrix [18].

VaR estimation for portfolio with normal distribution. To find risk measure VaR for normal distribution of risk factors at given confidence level and known portfolio cost it is necessary to compute standard deviation of return rate:

$$VaR = \alpha \sigma P_0 . \quad (7)$$

At the first step it is determined portfolio return rate, R_p, that is linear function of return rates of its components:

$$R_p = \sum_{i=1}^{N} \omega_i R_i \,, \tag{8}$$

where, N is number of portfolio components; R_i is return rate for i-th component; $w_i = P_i/P_p$ is weighting coefficient for i-th component; where, P_p is portfolio cost; P_i is cost of ith portfolio component. The matrix form of portfolio return rate is as follows:

$$R_p = [\omega_1, \omega_2, \dots, \omega_N] \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_N \end{bmatrix} . \tag{9}$$

For the vector of weighting coefficients, w, and vector of return rates, R, we have: $R_p = w^T R$. The next step is determining standard deviation for return, σ_p. Normal distribution of linear sum of normal random values gives normal distribution for the portfolio return rate, R_p. Thus, expected return rate is determined as follows: $\mu_p = \sum_{i=1}^{N} \quad w_i \mu_i$, and variance of the rate has the expression:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{i=1}^{N} \omega_i \omega_j \, \sigma_{ij} = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1, j<1}^{N} \omega_i \omega_j \, \sigma_{ij} \,, \tag{10}$$

where, σ_{ij} are elements of covariance matrix.

Properties of empirical correlation matrices. The correlation matrix is widely used in theoretical studies but in systems of risk management its estimate is used in the form of empirical correlation matrix. The model of averaged correlation in which all element are equal, ρ, but for the "1s" on main diagonal, there exists one large eigenvalue, $\lambda_1 = 1 + (N-1)\rho$, and all others eigenvalues are equal to, $\lambda_{i \geq 1} = 1 - \rho$. Similar result was found in the case when non-diagonal elements of correlation matrix are random values with mathematical expectation, ρ, and standard deviation, σ:

$$E[\lambda_1] = (N-1)\rho + \frac{\sigma^2}{\rho} + 1 + o(1) \,. \tag{11}$$

Thus, when, $\rho > 0$, maximum eigenvalue is increasing with increasing system dimensionality, N. To the dominating eigenvalue corresponds uniformly distributed on components eigenvector, $v_1(1/\sqrt{N})$. This vector has economic sense as a factor of influence on all risk positions or generalized market index. The factor can be used to explain large scale market crises. Such interpretation can be found in the studies of empirical financial correlation matrices [12, 19].

The study [19] on empirical matrices of linear correlation for 406 stock rates in the period of 1991-1996 showed correspondence between distribution of eigenvalues to theoretical results from the theory of random matrices, but for 6% of maximum eigenvalues. The work [10] points out to availability of concentration of particularly large eigenvalues for random symmetric matrices. In practice of computing experiments it was observed availability of several eigenvalues in the range that exceeds for about 5-10 times the basic bulk of eigenvalues. This situation can be explained by availability in the market besides basic generalized market factor other factors, say sector factors that influence some positions.

The study [7] proposed the group model of fund markets. According to the model assumptions the market includes several separate groups that use assets the prices of which correlate with prices of other assets of this group. In this situation the correlation matrix becomes close to the block-diagonal form where each block corresponds to the sector of economy with higher correlations in the frames of the block and lower correlations outside of the block. Close to this case is correlation matrix with, $N_1 \times N_1$, diagonal blocks, in the frames of which the correlation coefficients are equal, ρ_1, with "1s" on diagonal, and ρ_0, outside of the blocks. Then maximum eigenvalue of the correlation matrix is estimated as follows:

$$\lambda_1 = 1 + (N_1 - 1)\rho_1 + (N - N_1)\rho_0 \,, \qquad (12)$$

the eigenvalues that correspond to the eigenvectors that characterize principal factors influencing the branch of economy can be found as follows:

$$\lambda_1 = 1 + (N_1 - 1)\rho_1 + (N - N_1)\rho_0 \,, \qquad (13)$$

and other eigenvalues can be found as follows:

$$\lambda_{i=\frac{N}{N_1}+1 \ldots N} = 1 - \rho_1 \,. \qquad (14)$$

The matrices of correlation coefficients that include (but for coefficient of linear correlation because of its drawbacks regarding risk management) the concordance measures, were studied with the methods of random matrix theory [13]. It was pointed out in [12] to correspondence of results received for random symmetric matrix to distribution of distances between eigenvalues of empirical matrix of linear correlation for 1000 stocks of American companies for two-year period.

The matrices of Pearson, Kendall, and Spearman correlation coefficients are symmetric and that is why the case of maximum statistical independence was considered corresponding to the symmetry condition. The deviations from foresights of the random matrix theory indicate to existence of dependences characteristic for a specific system.

Methods

Numerical modeling using copulas. The complex structure of multivariate risk distributions makes direct analytical estimation of risk measures difficult. Therefore, numerical simulation methods, particularly Monte Carlo simulation, are used to estimate risk measures and perform scenario analysis.

In the proposed approach, marginal distributions describe the behavior of individual risk factors, while copulas define the dependency structure between them. This allows the model to generate multivariate samples that preserve both individual characteristics of risks and their joint behavior.

The generated samples are then used to estimate portfolio risk measures and analyze possible scenarios of market behavior. Such an approach is especially useful when dependencies between risk factors are nonlinear or become stronger in the tails of distributions.

In the multivariate case, sample generation can be performed either sequentially for each variable or directly for the whole multivariate distribution. The second approach is more suitable for modeling complex dependency structures, since it allows the joint behavior of all risk factors to be considered simultaneously.

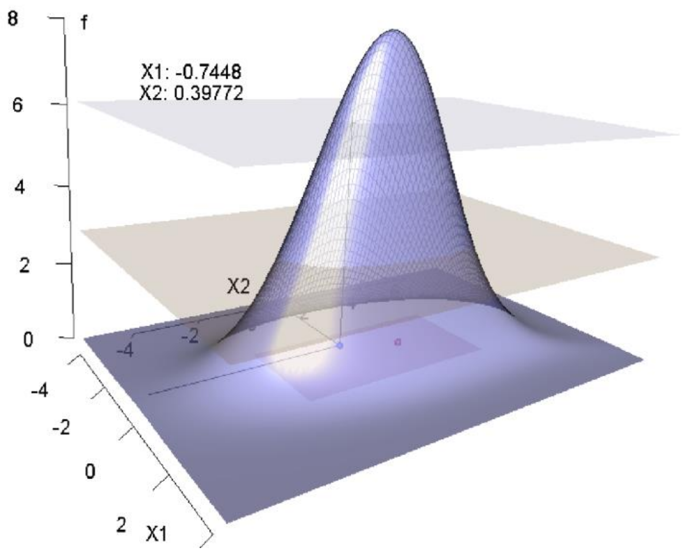

Fig. 2. Multivariate generalization of the generating method using cross-section

Application of models to actual data. Modeling of joint probabilistic distribution of risk factors was performed for simulated three-dimensional distributions of Cauchy, Student, normal, and exchange rate of currencies (with 15 min observation interval): EUR, CHF, GBP with respect to USD from 2009 to 2016. For each dataset was performed estimation of Archimedean copulas from the families: Gumball, Clayton, Frank and elliptical copulas from the Student family and normal distribution.

Together with estimates of marginal distributions this experiment provided the possibility for modeling the functions of joint distribution. Figs. 3–7 illustrate joint distributions for the currency exchange rates modeled with the use of various dependency structures.

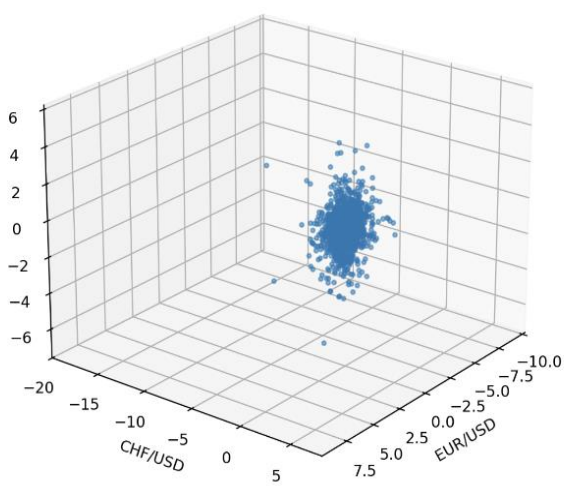

Fig. 3. Joint distribution on the basis of normal copula

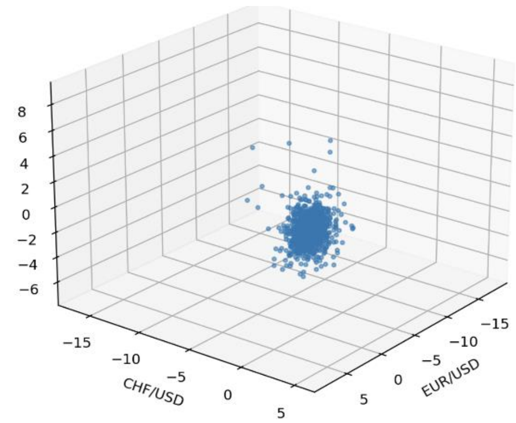

Fig. 4. Joint distribution on the basis of *t*-Student copula

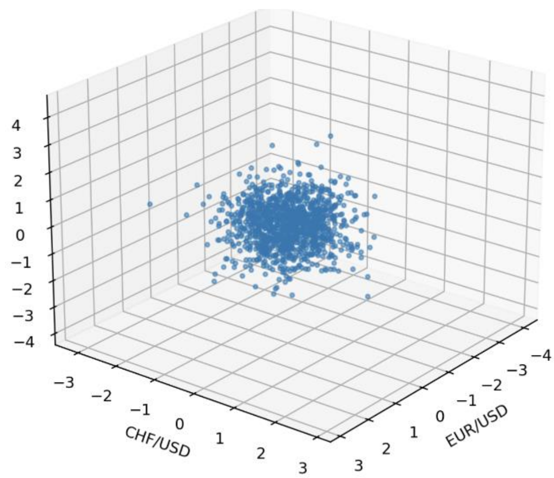

Fig. 5. Joint distribution on the basis of Frank copula

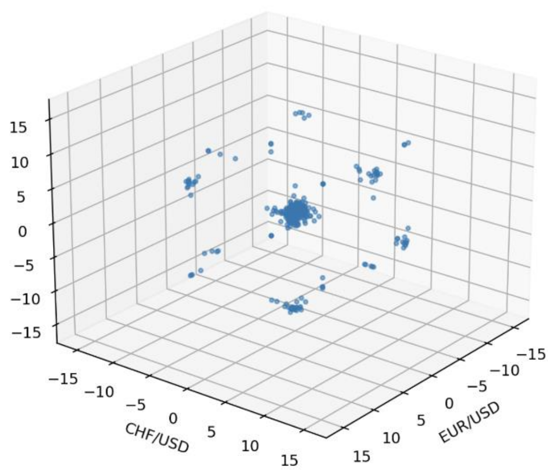

Fig. 6. Joint distribution on the basis of Gumball copula

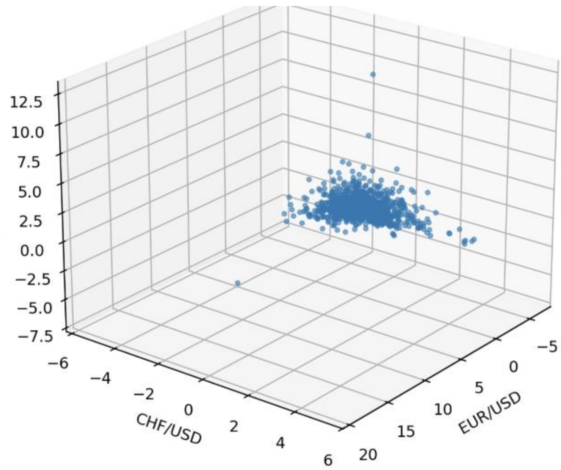

Fig. 7. Joint distribution on the basis of Clayton copula

Empirical joint distribution for the currencies mentioned is shown in Fig. 8.

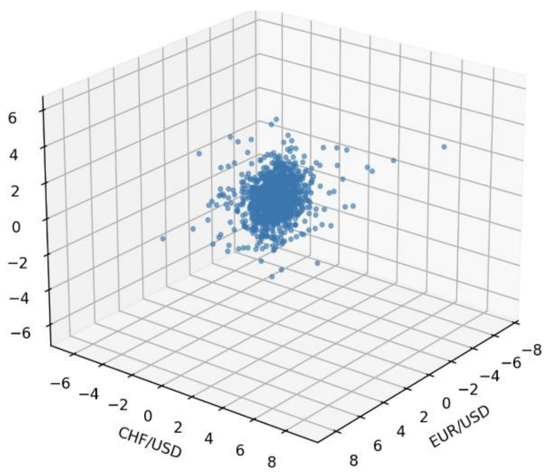

Fig. 8. Empirical joint distribution the currencies exchange rate

Results

The quantitative measures of risk based on risk measures and qualitative based upon scenario analysis provide researchers and risk managers with the integral picture regarding level of risk of available positions and portfolios [26]. An active risk management (control) requires constructing models that would allow to measure risk and determine its acceptability for organization as well as establish the level of risk by changing the structure of portfolio.

Estimation of risk through the market cost should also take into consideration the liquidity risk. This component comes to being through impossibility to sell an asset at definite moment using its market value. The liquidity risk is available on almost all financial and commodity markets. On the markets with low volumes of trading and during financial crisis the liquidity share reaches, 25%-30%. Volatility of liquidity imposes its restrictions on possible changes of portfolio structure during active phase of risk control. An active risk control requires develop the approach that would allow optimize with respect to estimates of risk measures the portfolio structure under restrictions of liquidity with respect to separate portfolio positions.

To solve the problem of active control for portfolio risks it was proposed to use the probabilistic-statistical model on the basis of combined distribution from normal and generalized Pareto distribution, marginal distributions of separate positions and multivariate distributions of portfolio loss. The multivariate distributions of loss are constructed by linking marginal distributions using special link functions in the form of copulas, and estimating risk measures related to tails and central parts of distributions.

The problem of finding optimal portfolio structure with respect to the VaR measure can be considered as the problem of optimizing estimate of VaR measure using the model with restrictions that reflect volatility of market liquidity. To estimate risk measures using the generated from the model of multivariate sample the cost of separate positions were found corresponding values of portfolio cost. If $\{X_{i:j}\}$ is sample of costs for, n-dimensional portfolio, and reordered in the way that, $X_{1:n} \leq .. \leq X_{n:n}$, then empirical estimate of VaR is the following:

$$VaR_{\alpha}(X) = X_{\max(i \in N | i \leq n\alpha):n^*} . \tag{15}$$

In the computing experiment were used daily exchange rates of Swiss franc, GB pound, Japanese yen and USD with respect to euro for nine years. Power of the sample was 1643 observations after preliminary data processing. The parameters of one-dimensional marginal distributions for each currency and copula parameters were estimated using the method of maximum likelihood.

Table 1. Estimates of copula parameters

Copula	Parameter	Value	MSE
Gumball	θ	1.6720	0.0158
Normal	ρ_1	0.5637	0.0118
	ρ_2	0.3318	0.0136
	ρ_3	0.5943	0.0120
	ρ_4	0.8241	0.0054
	ρ_5	0.8593	0.0051
	ρ_6	0.8037	0.0061
Frank	β	4.5874	0.0911

Empirical estimate of risk measure VaR with quintile, 0.03, i.e. for 50 observations that exceed the threshold is 3.497. For quintile, 0.01, i.e. for 16 observations that exceed the threshold the measure is 3.535. For quintile 0.03 there is enough observations to have a possibility for use of empirical estimate; for quintile, 0.01 the sample is too short and there is necessity to model risk distribution and estimation of risk with the model.

Table 2. Estimates of the risk measure VaR

Copula	Quintile	Power of sample		
		100	1000	10000
Gumball	0,03	3,4140	3,4770	3,4896
	0,01	3,4375	3,5665	3,6008
Normal	0,03	3,5006	3,5386	3,5346
	0,01	3,6177	3,6880	3,6603
Frank	0,03	3,4986	3,4797	3,4959
	0,01	3,5139	3,5632	3,5892

According to Table 2 the estimates of VaR measure using the models based upon combined marginal distributions and linked to joint distribution with the Gumball and Frank copulas have an error of about, 0.203%, and 0.022%, with respect to the empirical value for quintile, 0.03. The model with normal copula has an error of about, 1%. The results achieved provide the possibility for making conclusion that all three models are adequate and have possibility for their practical application. Thus, an estimate of the risk measure VaR can be considered 3.5892 for quintile 0.01. The same three models were used for estimating coherent risk measure ES (Expected Shortfall).

Table 3. Estimates of the risk measure ES

Copula	Quintile	Power of sample		
		100	1000	10000
Gumball	0.03	3.5121	3.5752	3.5835
	0.01	3.5348	3.6479	3.6737
Normal	0.03	3.6104	3.6475	3.6438
	0.01	3.7124	3.7733	3.7523
Frank	0.03	3.5857	3.5662	3.5836
	0.01	3.6017	3.6548	3.6822

Empirical estimate of ES with quintile 0.03 is 3.6074. The estimates in Table 3 show that the most adequate model for estimating this measure of risk is the model with Frank copula. Thus the model estimate for ES with quintile 0.01 is 3.682.

All three models showed worse results for the Markowitz deviation measure comparing to the empirical result. The models proposed are also used for active risk control through changing portfolio structure to optimize selected measure of risk.

Table 4. Estimates of the Markowitz risk measure

Copula	σ_+
Gumball	0.1330
Normal	0.1462
Frank	0.1370
Empirical estimate	0.1733

The constructed four-dimensional probabilistic distribution model was used to develop active risk control methodology by finding optimal portfolio structure on the basis of selected tail risk measure VaR. Fig. 9 illustrates the values of VaR estimates for various relationships between portfolio positions and liquidity restrictions that allowed for changings in positions of Swiss franc and GB pounds.

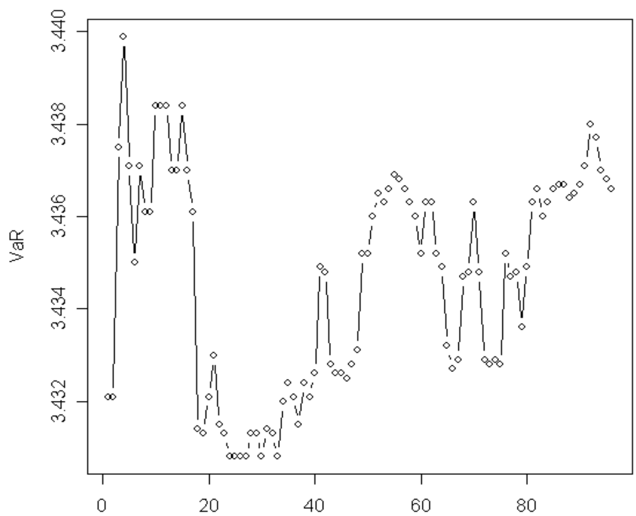

Fig. 9. An estimate of risk measure VaR depending on relation between positions in Swiss franc and GB pounds (accepted as 1); with step 0.1

It was also established that the values of tail risk measures for multivariate models of financial data exhibit nonlinear character with multiple local extremes depending on the portfolio structure. This character of dependency requires development effective optimization algorithm.

Table 5 demonstrates that empirical correlation matrices contain several dominant eigenvalues that substantially exceed the remaining part of the spectrum. These eigenvalues indicate the presence of key latent factors that determine the dependency structure of the analyzed financial instruments. The similarity of results for Kendall, Pearson and Spearman measures confirms the stability of the detected dependency pattern. At the same time, the remaining eigenvalues are close to the random-matrix range, which makes it possible to separate meaningful risk factors from noise.

Table 5. Maximum eigenvalues of empirical correlation matrices

Name	λ_1	λ_2	λ_3	λ_4
Kendall	236,2	60,4	24,4	14,8
Pearson	318,7	69,6	16,6	12,5
Spearmen	313,4	70,0	17,7	13,0

For example, for the matrix of linear correlations, $\lambda_{max} = 8.847487$, and four maximum eigenvalues are: 318.7, 69.6, 16.6 and 12.5, other 480 eigenvalues are positive and less than 1.02.

For empirical correlation matrices was estimated empirical distribution of distances between eigenvalues expanded with Gaussian unfolding and theoretical distribution of distances for corresponding symmetric random matrix from (5). The empirical distributions for correlation matrices and theoretical distributions for random matrices turned out to be similar for majority of eigenvalues except for maximum eigenvalue of empirical correlation matrix which turned out to be substantially larger than proposes theoretical distribution.

For linear correlation it corresponds to the level of about, 99.9992%. In right tail of distribution theoretical threshold for 95% of observations exceed 5% of eigenvalues; and the threshold of 97% exceed 2.7% of eigenvalues (Fig. 10).

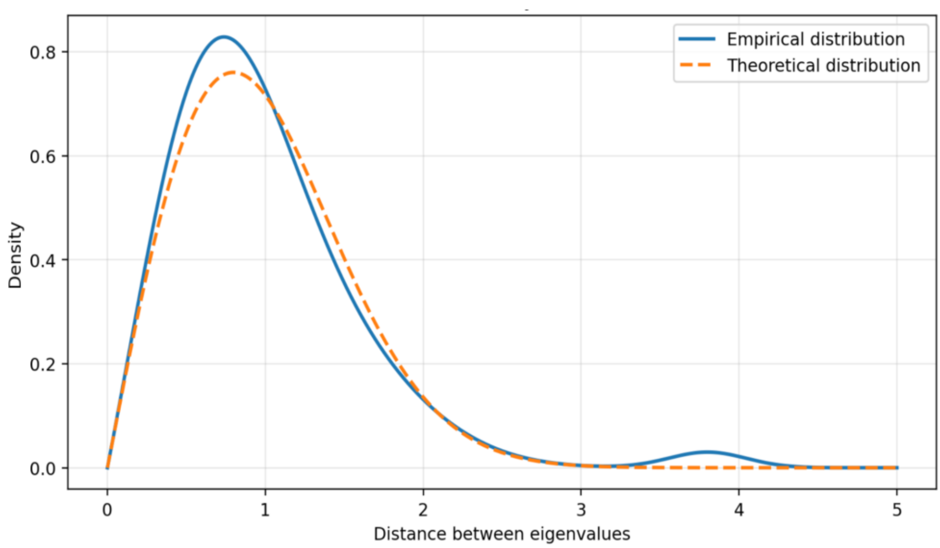

Fig. 10. Empirical density distribution of distances between eigenvalues of empirical matrix of linear Pearson correlation coefficients and theoretical density of distances distribution

The distributions of eigenvalues and distances between eigenvalues for empirical correlation matrices and different dependency measures used in the experiment demonstrate similar behavior. The eigenvalue distributions provide the possibilities for determining the number of basic factors in the model.

Discussion

The number of risk factors in large scale financial systems can be very high, and that is why the problem of multivariate risk analysis attracts attention of scientists, engineers and risk managers the world over. Interaction and simultaneous influence of multiple risk factors in high-dimension systems can result in substantial increase of total loss comparing to the cases when interaction between elements of the systems is taken into consideration.

The main task of the studies is to identify principal factors making substantial influence on the value of possible loss. The theory of random matrices provides the possibilities for analysis of eigenvalues distribution regarding correlation matrices of dependency measures. The results of various studies show that this is the possibility for receiving practically useful information to be further used in management of multivariate risks. It is possible to carry out the studies in the future directed to improvement of the results using theoretical distributions of eigenvalues and distances between eigenvalues for symmetric positively defined matrices. Also has perspective analysis of an influence of non-linear strictly increasing transforms on distribution of eigenvalues of dependency measures for empirical matrices.

Conclusions

The proposed system-analysis approach enables modeling dependencies between risk factors using matrices of dependency measures and copula families with estimated parameters. This makes it possible to construct a multivariate risk model in which marginal distributions and dependency structures are modeled separately. Methods for estimating copula parameters were considered, including a two-step maximum likelihood procedure for joint distribution modeling. The results confirm that this approach can be applied to practical risk management tasks and scenario analysis. The study also used mutual information within a Bayesian network framework to determine risk dependencies and account for expert knowledge and new information during risk management. The analysis of Pearson, Kendall and Spearman correlation matrices showed that dominant eigenvalues exceed theoretical random-matrix limits. This indicates the presence of key latent factors, while smaller eigenvalues mainly correspond to noise. Therefore, portfolio optimization based on Markowitz theory should be performed after filtering noisy data. Computational experiments with generated and empirical three-dimensional distributions confirmed the applicability of the proposed approach to multivariate risk modeling. For tail risk estimation, the proposed model supported portfolio structure optimization under liquidity constraints. The estimation error for non-extreme quantiles was less than one percent, while tail and deviation risk measures require further model improvement.

REFERENCES

1. Jorion, P. (2003). Financial Risk Manager Handbook New Jersey: Wiley.
2. Bidjuk, P., Timoshhuk, O., Gus'kova, V., Prosjankina-Zharova, T. & Levenchuk, L. (2025). Analiz nestacionarnih ta nelinijnih procesiv v ekonomici ta finansah. Kiïv: KPI im. Igorja Sikors'kogo (in Ukrainian). URL: https://ela.kpi.ua/handle/123456789/77009
3. Fenton, N., Neil, M. (2013). Risk Assessment and Decision Analysis with Bayesian Networks. Boca Raton, Fl: CRC Press.

4. Turban, E. & Aronson, J. E. (2001). Decision support systems. New Jersey: Prentice Hall.
5. Dovgij, S. O., Bidjuk, P. I. & Trofimchuk, O. M. (2014). Sistemi pidtrimki prijnjattja rishen' na osnovi statistichno-jmovirnisnih metodiv. Kiïv: Logos. (in Ukrainian). URL: https://nvd-nanu.org.ua/en/c852c5e1-c193-1c64-fb13-67b818323660/
6. Malevergne, Y. & Sornette, D. (2004). Collective origin of the coexistence of apparent RMT noise and factors in large sample correlation matrices. “Physica A: Statistical Mechanics and its Applications”, 331, 3-4, 660–668. https://doi.org/10.1016/j.physa.2003.09.004
7. Noh, J. (2000). A model for correlations in stock markets. “Physical Review E”, 61, 5981–5982. https://doi.org/10.1103/PhysRevE.61.5981
8. Fama E. & French K. (1996). Multifactor explanations of asset pricing anomalies. “Journal of Finance”, 51, 1, 55–84. https://doi.org/10.1111/j.1540-6261.1996.tb05202.x
9. Mehta, M. (2004). Random Matrices. Amsterdam: Elsevier Academic Press.
10. Alon, N., Krivelevich, M. & Vu, V. H. (2002). On the concentration of eigenvalues of random symmetric matrices. “Israel Journal of Mathematics”, 131, 259–268. https://doi.org/10.1007/BF02785860
11. Laloux, L., Cizeau, P., Potters, M., & Bouchaud, J. (2000). Random matrix theory and financial correlations. “International Journal of Theoretical & Applied Finance”, 3, 391–397. https://www.cfm.com/wp-content/uploads/2022/12/234-1999-random-matrix-theory-and-financial-correlations.pdf
12. Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral L. & Stanley, H. (1999). Universal and Nonuniversal Properties of Cross Correlations in Financial Time Series. “Physical Review Letters”, 83, 7, 1471-1474. https://doi.org/10.1103/PhysRevLett.83.1471
13. Sharifi, S., Crane, M., Shamaie, A. & Ruskin H. (2004). Random matrix theory for portfolio optimization: a stability approach. “Physica A: Statistical Mechanics and its Applications”, 335, 3-4, 629-643. https://doi.org/10.1016/j.physa.2003.12.016
14. Nelsen, R.B. (2006). An Introduction to Copulas. New York: Springer, 2006.
15. Granger C., Maasoumi E., & Racine J. A. (2004). Dependence Metric for Possibly Nonlinear Processes. “Journal of Time Series Analysis”, 25, 5, 649-669. https://doi.org/10.1111/j.1467-9892.2004.01866.x
16. Kurowicka, D. (2006). Uncertainty Analysis with High Dimensional Dependence Modelling. New Jersey: Wiley.
17. Markowitz, H. Portfolio Selection (1952). “Journal of Finance”, 7, 77-91. URL: https://finance.martinsewell.com/capm/Markowitz1952.pdf
18. Pafka, S. & Kondor, I. (2004). Estimated correlation matrices and portfolio optimization. “Physica A: Statistical Mechanics and its Applications”, 343, 623–634. https://doi.org/10.1016/j.physa.2004.05.079
19. Laloux, L., Cizeau, P., Bouchaud, J.-P. & Potters, M. (1999). Noise Dressing of Financial Correlation Matrices. “Physical Review Letters”, 83, 7, 1467-1470. https://doi.org/10.1103/PhysRevLett.83.1467
20. Sastry, S., Deo, N. & Franz, S. (2001). Spectral Statistics of Instantaneous Normal Modes in Liquids and Random Matrices. “Physical Review E”, 64, 16305–16309. https://doi.org/10.1103/PhysRevE.64.016305
21. Bruus, H., Angles & d’Auriac J.C. (1997). Energy level statistics of the two-dimensional Hubbard model at low filling. “Physical Review B”, 55, 9142–9159. https://doi.org/10.1103/PhysRevB.55.9142
22. Li, David X. (2000). On Default Correlation: A Copula Function Approach. “Journal of Fixed Income”, 9, 43–54. http://dx.doi.org/10.2139/ssrn.187289
23. Sengupta, A. M. & Mitra, P. P. (1999). Distributions of singular values for some random matrices. “Physical Review E”, 60, 3, 3389–3392. https://doi.org/10.1103/PhysRevE.60.3389
24. Longin, F. M. (2000). From value at risk to stress testing: The extreme value approach. “Journal of Banking and Finance”, 24, 1097–1130. https://doi.org/10.1016/S0378-4266(99)00077-1

25. Scaillet, O. (2007). Kernel based goodness-of-fit tests for copulas with fixed smoothing parameters. "Journal of Multivariate Analysis", 98, 533-543. http://dx.doi.org/10.2139/ssrn.731784
26. Huckins, N.W. & Rai, A. (1999). Market Risk for Foreign Currency Options: Basle's Simplified Model. "Financial Management", 28, 1, 99-109. URL: https://ideas.repec.org/a/fma/fmanag/huckins99.html

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АНАЛІЗ ФІНАНСОВИХ РИЗИКІВ У БАГАТОВИМІРНИХ СИСТЕМАХ

Анотація. Запропоновано новий підхід для моделювання залежності між факторами багатовимірних ризиків, представлених у вигляді матриць мір залежності для чисельного опису і сімейства копул з оцінками їх параметрів для аналітичного опису, що дало можливість побудувати багатовимірну модель ризиків, за якої, окремо моделюються маргінальні розподіли з використанням еліптичних розподілів для вимірів у центрі вибірок та екстремальні розподіли у хвостових частинах, залежності між ризиками моделюються копулами. Спільний розподіл моделюється за допомогою маргінальних розподілів і копул, може бути застосований для аналізу характеристик ризиків. Розроблено підхід до визначення залежності ризиків з використанням поняття взаємної інформації в межах побудови байєсівських мереж. Обчислювальний експеримент з двома згенерованими, відомими з точки зору теорії, тривимірними розподілами та одним емпіричним тривимірним розподілом для курсів обміну валют продемонстрували можливість застосування запропонованого підходу до моделювання багатовимірного ризику.

Для вирішення проблеми пошуку структури оптимального портфеля в умовах активного керування ризиками та обмежень на ліквідність активів, запропоновано багатовимірну модель для оцінювання хвостових мір ризику. Обчислювальний експеримент, виконаний для оцінювання мір ризику шляхом генерування вибірки, забезпечив похибку оцінювання, меншу одного процента для неекстремальних квантилів. Якість оцінювання мір відхилення ризику вимагає подальшого удосконалення моделі. Якість оцінювання мір ризику для хвостових частин розподілів свідчить, що модель на основі комбінації маргінальних розподілів з використанням нормального і Парето розподілів потрібно покращити для опису центральних спостережень.

Ключові слова: системний аналіз, фінансові ризики, багатовимірний розподіл, сімейства копул, спільний розподіл, міри залежності, комбінований маргінальний розподіл.

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