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Oleksandr Trofymchuk, Corresponding Member of the NASU, D. S. (Computer science), Professor, Director of the Institute of Telecommunications and Global Information Space
ORCID ID: https://orcid.org/0000-0003-3358-6274 ***e-mail:*** itgis@nas.gov.ua

Volodymyr Vasyanin, D. S. (Computer science), Head of department
ORCID ID: https://orcid.org/0000-0003-4046-5243 ***e-mail:*** archukr@meta.ua

Institute of Telecommunications and Global Information Space of the National Academy of Sciences of Ukraine, Kyiv, Ukraine

A TWO-STEP PROBLEM OF OPTIMIZING THE STRUCTURE AND ROUTING OF FLOWS IN A HIERARCHICAL MULTICOMMODITY NETWORK

Abstract. *The paper discusses the methodology of mathematical modeling of the two-stage problem of optimization of the backbone hierarchical communication network with multicommodity discrete flows and parameters. The methodology is based on the sequential solution of the problem of optimizing the network structure and the problem of distribution and routing of discrete correspondence flows. As a rule, such networks consist of a decentralized backbone network and fragmented networks in the internal service areas of the backbone nodes. There are four types of network nodes and three levels of its hierarchy. In a multicommodity network, each node can exchange correspondence (products, goods, cargo, messages) with other nodes. Correspondence is characterized by a source node, a drain node and a value, which for transport networks is given by the number of packaged goods, cargo in a package of a unified size, and for data transmission networks – by the number of bytes, kilobytes, etc. In the transport backbone network, all correspondence is first sorted by destination addresses, packed in transport blocks (containers), and then transported in vehicles along the transport highways. In data networks, correspondence is also sorted by destination addresses (multiplexed), packaged in virtual transport blocks, and then transmitted over trunk communication channels. The size (capacity, volume) of the transport blockt is set by the parameter, and is determined by the number of units of correspondence that fit into it. Mathematical models of problems of optimization of network structure, distribution and routing of flows, and an example of numerical modeling of solving problems on a transport network containing 120 nodes and 300 unoriented arcs are presented. Experimental studies have shown high computational efficiency of the proposed algorithms and programs, and they can be recommended for the practical solution of problems of optimizing the processes of processing and transporting flows in communication networks of large dimensions.*

Keywords: *multicommodity hierarchical networks, discrete flows, problems of combinatorial optimization, mathematical models, computer modeling.*

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Introduction

As a rule, existing and projected communication networks have a hierarchical structure and consist of a decentralized backbone network and networks in the internal service areas of the backbone nodes (internal networks). One of the most important tasks included in the mathematical support of the information-analytical system (IAS) for managing the processes of processing and distribution of discrete flows in a multi-product hierarchical network is forecasting and optimizing the phased development of its infrastructure. At the same time, it is necessary to take into account all the possibilities of high-quality organizational and technical improvement of the network in the presence of restrictions on capital investments (investments) and the possibility of their development. In fact, the solution of these tasks determines the plans for the commissioning of production facilities and fixed assets of network elements (nodal enterprises and communication lines – nodes and routes of vehicles in the transport network, nodes and communication channels in the data transmission network) and labor resources by stages of development. As a rule, long-term development plans are built for several years (from three to ten), and one year acts as a planning stage (planning time discrete). The results of solving such large-aggregated problems – the distribution of capital investments and other material resources by stages of implementation – should be used in the construction of more detailed models at each stage. So, for example, for transport networks, it is necessary to divide the entire amount of capital investments of the stage into the purchase, modernization and commissioning of sorting equipment (cargo sorting lines), equipment for loading and unloading goods in junction enterprises, vehicles for existing and new transportation routes. For data transmission networks, it is necessary to divide the financial resources of the stage into modernization and re-equipment between nodes (multiplexers, switches, routers, etc.) and communication channels.

In each specific case, the choice of the duration of the stage of prospective forecasting is an independent task and depends on the speed of introduction and development of new equipment and information technologies in the network infrastructures of various industries in the economic sphere. The stability of the functioning of network structures during the planning stages in case of load fluctuations in network nodes and communication lines, the occurrence of failures and the action of random factors should be ensured by the tasks of operational management. An extensive bibliographic overview of mathematical models, methods, and algorithms for solving Multicommodity Network Flow Problems (MCNF) can be found in [1-8]. One of the few papers [9] considers the problem of routing groupage packaged cargo in a multi-product network, in which the processes of their sorting and transportation are integrated and restrictions on the time of delivery of goods are taken into account. At the same time, only such cargoes are grouped into one transport block, in which the points of departure and destination and time windows of delivery coincide, restrictions on the capacity of nodes and arcs of the network are not taken into account. [10, 11] presents multi-product routing models with constraints on the bandwidth of network arcs and with hard (the Hard Transit Time-Constrained, HTC-MCNF) and soft (the Soft Ttransit Ttime-Constrained, STC-MCNF) constraints on the delivery time of goods, but cargoes with different destination addresses are not combined into common transport blocks.

Unlike most existing approaches to modeling and analyzing the functioning of multi-product networks, this paper considers discrete models of transport processes with integer variables and parameters. Practical tasks should take into account the processes of sorting goods in sorting centers, restrictions on the time of their delivery to the consumer, fluctuations in flows and loads in individual nodes and communication lines of the transport network, carrying capacity of vehicles, nonlinearity of the given costs for processing and transportation of flows and many other real factors and constraints. This leads to the need to develop new mathematical models, methods, algorithms and information platform for managing the handling, distribution and routing of groupage cargo flows and determines the importance of the studied scientific and applied problem for the development of the transport system of Ukraine. This work is a continuation of a series of studies of hierarchical networks with discrete flows [12-23], which proposes a methodology for mathematical modeling of the phased development of nodes and transport routes of the backbone network, based on solving the problems of optimizing its structure and distribution of flows.

1. Hierarchical Network Structure

Each node in a hierarchical network has a name, a unique index, and a sequence number. Each node can be matched with a set of indices (numbers) of other nodes corresponding to it in the backbone and internal network. In a multi-product network, each node can exchange correspondence (messages, cargo) with all other nodes. Correspondence is characterized by a source node, a drain node and a value, which for data transmission networks is given by the number of bytes, kilobytes, etc., and for transport networks – by the number of packaged goods in a package of a unified size. In the backbone network, all correspondence is transmitted via communication channels or transported in vehicles in transport blocks of a given size (capacity, volume). The size of the transport unit is measured by the number of units of correspondence that fit into it (for example, 64 KB, 40 packaged cargo). All trunk nodes are sorting centers in which correspondence is first sorted by destination addresses (nodes) and then packed into transport blocks. In data networks, data transmission multiplexers play the role of sorting centers, and a virtual container acts as a transport unit. Since the size of individual correspondence is much smaller than the size of the transport block, they can be combined (packed) several times and in different nodes with correspondence with different destination addresses during sorting (multiplexing). With such a consolidation of correspondence in the network nodes, the number of directions for their sorting and the number of transport blocks required for their packaging are reduced, but in some nodes there are additional volumes of sorting correspondence that have not reached the address of their destination. In addition, the time of delivery to the recipient of correspondence, which undergoes additional sorting at transit hubs, increases.

In the inner zone of each trunk node there are nodes for the delivery and collection of correspondence (clients of the data transmission network or transport network), which can exchange correspondence with each other and with other nodes of the hierarchical network only through this trunk node. The transmission or transportation of correspondence in the internal network is carried out by regional providers or along the routes of internal vehicles.

Fig. 1 shows fragments of a three-tier network, where i, j, k – trunk nodes with their own backbone service zones (SZ), m – nodes of delivery and collection of correspondence in the internal service area (SA) of each trunk node (internal networks).

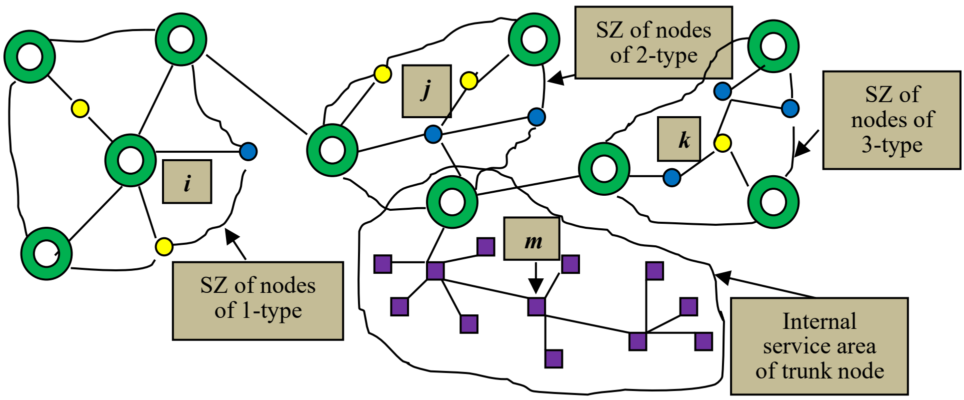


Fig. 1. Fragments of a hierarchical network

Trunk nodes of different types differ in functionality, level of technical equipment, number of service personnel, etc. Some of them can sort flows to all trunk nodes, while others can only sort to trunk nodes in their service area. In some trunk hubs, it may be prohibited to sort transit correspondence flows and process transit flows of transport blocks.

In [12] discusses the generalized problem of packaging and distribution of correspondence flows in a hierarchical network, the solution of which is carried out in several stages. At the first stage, the problem of choosing the hierarchical structure of the backbone network and the scheme of sorting correspondence in the network nodes and packing them into transport blocks is solved [14-16, 19]. At the second stage, there is a problem of distribution and routing of flows of transport blocks with collective correspondence, which were formed when solving the first problem [13, 17, 18, 20, 21, 22-25]. Groupage correspondence means messages combined into one transport block or packaged cargoes with different addresses of destination, which may not coincide with the address of destination of the transport block. Groupage correspondence is formed to minimize the number of transport units required for their packaging and transfer or transportation in the backbone network.

2. Problems of Correspondence Packaging and Network Structure Selection

For futher network development, according to the input data, it nessesary to solve the problem of optimizing its structure, to obtain a scheme for sorting outgoing flows and distributing the costs of sorting flows and the costs of loading and unloading outgoing and incoming transport blocks by nodes. The general scheme of the algorithm for solving the problem is based on the solution of the packaging problem [14, 15]. The problem of packaging arises in trunk transport networks and backbone data transmission networks with discrete flows of correspondence from suppliers to recipients, the essence of which is to concentrate the flows of cargo and information between network nodes. Correspondences with different destination addresses emanating from network nodes can be combined with each other and packed into common transport blocks (containers or virtual containers). As a result of the merging of flows, the number of transport blocks for the transportation of correspondence is reduced, the number of directions for sorting correspondence in the network nodes to other nodes is reduced, the load factor of transport blocks and vehicles increases, and the high capacity of the main communication channels is used more productively. At the same time, transit flows of correspondence arise in some nodes of the network, which leads to an increase in the total cost of processing flows in the network and an increase in the time of delivery of correspondence to the recipient.

Let $G(N,E)$ – hierarchical backbone network with set of nodes $N = N_1 \cup N_2 \cup N_3$, $n = |N|$, where N_1, N_2, N_3 – sets of nodes of the first, second and third type, respectively, and a set of undirected arcs E, $e = |E|$. It is assumed that the geographical coordinates of the location of the network nodes are known and the integer matrix of correspondence flows between all network nodes is known $A = \| a_{ij} \|_{n\times n}$, in which the rows correspond to the sending nodes and the columns to the destination nodes, and there is some transformed matrix $X = \| x_{ij} \|_{n\times n}$, whose elements are integer sought variables of the packaging problem. All correspondence is homogeneous (of the same type), and during transportation it can be combined in different nodes and packed in transport blocks only as a whole, that is, it is forbidden to branch them (splitting into parts) and transport them along several routes. Flows a_{ii}, $i = \overline{1,n}$, in the matrix A represent internal flows between nodes of the fourth type in the service area i-th nodes that are not transported through the backbone network. All correspondence in the backbone network must be transported in some fixed-capacity transport blocks $\omega >> a_{ij}$, $i, j = \overline{1,n}$, $i \neq j$, $\omega \in Z^+$.

It is required to determine the quantitative and qualitative composition of the nodes network and correspondence sorting scheme at each node, which minimizes the reduced costs of network operation. Formally, it is necessary to solve the packaging problem for all possible combinations of node types and find a minimum of functions

$$F = \sum_{ij\in S} C_{tr}^{ij}(u_{ij}, d_{ij}) + \sum_{i=1}^{n} C_{sort}^{i}(x_i, q_i) + \sum_{i=1}^{n} C_{load}^{i}(u_i) \qquad (1)$$

subject to

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1, j\neq i}^{n} a_{ij} = \sum_{j=1}^{n} x_{ji} - \sum_{j=1, j\neq i}^{n} a_{ji},\ i = \overline{1,n}, \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1, j\neq i}^{n} a_{ij} \leq h_i,\ i = \overline{1,n}, \qquad (3)$$

$$x_{ij}, u_{ij} \geq 0 \text{ and integers}, \forall\, ij \in S, \qquad (4)$$

when solving the problem, the constraints on the time of delivery of correspondence flows to the recipient are also taken into account

$$t_{ij} \le T_{ij} \quad \forall\, ij \in S, \tag{5}$$

and the number of transit merges

$$\nu_{ij} \le \nu_{\max} \quad \forall\, ij \in S, \tag{6}$$

where S – set of index pairs (i, j) corresponding nodes; $x_{ij} = a_{ij} + \sum_{rs} a^*_{rs}$, if the correspondence a_{ij} has not been combined with any other correspondence, where $\{a^*_{rs}\}$ – a set of correspondence combined with correspondence a_{ij}, and $x_{ij} = 0$, if the correspondence a_{ij} merged with any other correspondence or $i = j$; $C^{ij}_{tr}(u_{ij}, d_{ij})$ – nonlinear transport cost function dependent on the number of transport blocks $u_{ij} = \left\lceil \frac{x_{ij}}{\omega} \right\rceil$, ($\lceil x \rceil$ – smallest integer, greater than or equal x) and length d_{ij} – ways of their transportation between nodes i and j.

As a rule, in mathematical models describing the processes of processing and transporting flows, costs are associated with the value of the flow along the arcs of the network or the routes of flow transmission. For data networks, where arcs are associated with communication channels, such arrangements are quite acceptable. In the case of transport networks, it is much more difficult to adequately define cost functions, such as $C^{ij}_{tr}(u_{ij}, d_{ij})$, therefore, as a result of solving the problem, get a reliable answer. In problem (1)-(4), only the approximate estimate of transport costs and the lower limit of costs for processing transport blocks are calculated. Therefore, as a $C^{ij}_{tr}(u_{ij}, d_{ij})$ the function of the unit cost of transportation of the flow of the value of the u_{ij} at a distance d_{ij} on the carrying capacity of the vehicle or the bandwidth of the communication channel, which is set as a parameter $w_\xi \in \{w_1, w_2, ..., w_\alpha\}$, $\xi = \overline{1, \alpha}$. For example, it can be assumed that $C^{ij}_{tr}(u_{ij}, d_{ij}) = u_{ij}(k^\xi_1 + k^\xi_2 d_{ij}) / w_\xi$, $\forall\, ij \in S$, where k^ξ_1, k^ξ_2 – specified coefficients. At the same time, the structure of the network does not depend on the choice of value w_ξ, and transport costs are calculated only approximate (at a given value of w_ξ) in the distribution of the formed flows of transport blocks along the shortest paths according to the lexicographic criterion: minimum arcs in the path, minimum path length; $C^i_{sort}(x_i, q_i)$ – nonlinear cost function of total volume $x_i = a_{ii} + \sum_{j=1, j\ne i}^{n} (a_{ij} + a_{ji}) + \sum_{j=1}^{n} x_{ij} - \sum_{j=1, j\ne i}^{n} a_{ij}$ and the number of sorting directions $q_i = q^i_{in} + \sum_{j=1}^{n} \delta_{ij}$ correspondence processed in the node i ($\delta_{ij} = 1$, if $x_{ij} \ne 0$ and $\delta_{ij} = 0$, if $x_{ij} = 0$, and q^i_{in} determines the specified number of sorting directions for processing correspondence a_{ii}, $i = \overline{1, n}$); $C^i_{load}(u_i)$ – nonlinear cost function of total number of transport blocks $u_i = \sum_{j=1}^{n}(u_{ij} + u_{ji})$, processed in the node i; h_i, $i = \overline{1,n}$ – maximum throughput of the i-th node for processing transit correspondence. For nodes of the second and third types $h_i = 0$.

When solving the problem, delivery time constraints are also taken into account $t_{ij} \le T_{ij}$ $\forall\ ij \in S$ and the number of transit merges $\nu_{ij} \le \nu_{\max}$ $\forall\ ij \in S$, correspondence during their transportation from the nodes of departure to the nodes of destination, where T_{ij} and $\nu_{\max}$ – the time of delivery of correspondence to the recipient and the maximum allowable number of transit associations of correspondence are set accordingly. When calculating the delivery time, parameters are used that are not explicitly included in the model: the specified time for sorting correspondence and the time for transit overloading of transport blocks in network nodes, the average speed of vehicles or message transmission, etc. (the 4 part of this work).

The first component of function (1) determines the transport costs, the second – the sorting costs, and the third – the costs of handling the transport blocks. Expressions (2), (3), and (4) represent balance conditions, node bandwidth constraints, and variable values x_{ij}.

In [15] algorithms are proposed for solving the problem of packaging with functions of costs for processing and transportation of correspondence, which are based on a discrete analogue of the local descent method, when the vicinity of the metric space of admissible solutions is selected for heuristic considerations, taking into account the specifics of data structures and features of the problem.

It should be noted that when solving the problem, it is possible not to take into account the constraints on the bandwidth of nodes (3). Additional constraints $t_{ij} \le T_{ij}$ (5), $\nu_{ij} \le \nu_{\max}$ (6), $\forall\ ij \in S$ they may also not be taken into account, but at the request of the transport network designer or the administrator of the data network, all constraints can be set as directives. If the problem of choosing a structure was solved for the transport network, then it is necessary to additionally solve the problem of delivering empty containers [26].

In the process of solving the problem for network development, not only the structure of the network is determined, but also the optimal scheme for sorting the outgoing flows of correspondence in the selected structure. If the present cost functions are used $C^i_{sort}(x_i, q_i)$, $C^i_{load}(u_i)$, adequate to the processes of sorting and processing, then for the network nodes it is possible to obtain a fairly realistic estimate of their size, that is, to determine the necessary reduced costs for the functioning of the nodes for the period of completion of planning (the second component of the function (1) and partially the third – without taking into account the costs of transit transshipment of transport blocks). For transport costs and handling costs of transport blocks, only preliminary estimates are calculated. Real estimates of these costs can be obtained only after solving the problem of distribution and routing of the formed transport blocks on the transport network or data transmission network (only incoming and outgoing flows are taken into account in the costs of processing transport blocks in network nodes – transit flows are not taken into account; for transport networks, the costs of transportation and handling of empty containers are not taken into account).

In addition to the structure of the network, the main results of solving the problem are flow matrices $X = \| x_{ij} \|_{n \times n}$ and $U = \| u_{ij} \|_{n \times n}$ correspondence and transport blocks; matrix of preliminary estimates of the time of delivery of correspondence to recipients $\tilde{T} = \| \tilde{t}_{ij} \|_{n \times n}$; reference matrix of merging correspondence flows $C = \| c_{ij} \|_{n \times n}$ [27], the elements of which are defined as follows:

$$c_{ij} = \begin{cases} k, \text{ if flow } a_{ij} \text{ merges with the flow } a_{ik}, \\ i, \text{ if flow } a_{ij} \text{ is directly sent to the node } j, \\ 0, \text{ if } i = j, \end{cases}$$

where k – the node through which the flow conversion is performed a_{ij}. Matrix C it is used to restore the sequence of network nodes $\Omega_{ij} = \{(i, k_1), (k_1, k_2), ..., (k_m, j)\}$ with intermediate nodes $\{k_1, k_2, ..., k_m\}$, in which additional (transit) sorting of each of the flows is performed a_{ij}, $i, j = \overline{1, n}$, $i \neq j$, and their total number $\nu_{ij} = | \{k_1, k_2, ..., k_m\} |$, and calculation t_{ij} – the time of their delivery to the final recipients. These results are used as input data to solve the problem of distribution and routing of transport block flows. The reference matrix of flow merging fully defines the correspondence sorting scheme in all network nodes and addresses the flows of transport blocks that will be distributed along the routes of vehicles or communication channels. In the nodes of a real transport network or data transmission network, the reference matrix is used for automated control of equipment that carries out the processes of sorting address cargo, or as a table for merging messages into virtual containers.

3. The Problem of Distribution and Routing of Transport Block Flows

Based on the results of solving the problem of optimizing the network structure for its development, need to solve the problem of distribution and routing of flows. The problem of distribution and routing of discrete multicommodity flows of transport blocks with groupage (mixed, combined) correspondence arises in transport networks with discrete cargo and in backbone data transmission networks with the technology of virtual containers [13]. Groupage correspondence, as before, means packaged cargoes with different addresses of destination combined into one transport block (container) which addresses may not coincide with the address of destination of the transport block. The substantive formulation of the problem consists in the selection of such a scheme for the distribution and routing of flows of transport blocks, in which the reduced costs for processing and transportation of flows are minimized. The solution of the problem should be carried out in an interactive mode and determine the main technical and economic indicators of the backbone network functioning when changing the initial data, parameters and constraints of the transport model. Since it is not always possible to formalize all the factors influencing the choice of the best solution, the practical experience of transport network dispatchers and administrators of data transmission networks, as well as the knowledge base in the information and analytical decision support system [28], can be used for the final selection of the distribution and routing scheme of flows.

Let $G(N,P)$ – a hierarchical backbone network with a set of undirected arcs P, $p = |P|$ and a set of nodes $N = N_1 \cup N_2 \cup N_3$, $n = |N|$, where N_1, N_2, N_3 – sets of nodes of the first, second and third types, respectively. The nodes of the network correspond to the nodes of sorting, departure, destination and overload of flows, and the arcs correspond to the sections of roads for transport networks or communication channels for data networks connecting network nodes.

The flows of network are defined by the initial $A = \| a_{ij} \|_{n\times n}$ and transformed $A' = \| a'_{ij} \|_{n\times n} = X = \| x_{ij} \|_{n\times n}$, integer matrices of correspondence and integer matrix of transport blocks $\tilde{A} = \| \tilde{a}_{ij} \|_{n\times n} = U \| u_{ij} \|_{n\times n}$. There is also a matrix of preliminary estimates of the time of delivery of correspondence to recipients $\tilde{T} = \| \tilde{t}_{ij} \|_{n\times n}$ and other information obtained after solving the problem of choosing the network structure. Matrix elements $\tilde{T}$ act as initial constraints on delivery time when solving the problem of distribution and routing of flows of transport blocks.

Let $\{m_k\}$, $k = \overline{1,l}$ – a given set of projected vehicle routes or communication channels, each of which consists of a sequence of network nodes and arcs G, connecting the start and end nodes of a route or communication channel. It is assumed that the set $\{m_k\}$ for each undirected arc of the network G contains forward and reverse routes, and in the process of solving the problem in the set $\{m_k\}$ new routes generated by certain rules may be included. Set $\{m_k\}$ can contain multiple routes connecting any pair of nodes. Each route of the transport network is associated with its characteristics: the function of the average annual present costs for the operation and maintenance of the route; carrying capacity and frequency of movement of vehicles; time of arrival and departure of the vehicle for each node in the route, etc. For each route in the data transmission network, the function of the average annual reduced costs for the operation and maintenance of the communication channel, its length and bandwidth are specified.

We will build a route multi-network $G_M(N, P_M)$ transitive closure of nodes of all routes with $\{m_k\}$, where N – a set of network nodes, P_M – the set of its oriented route arcs. Between any nodes α and β network G_M there is a route arc if they are connected by at least one vehicle route or communication channel with $\{m_k\}$. Let's introduce the variables: $u_{ij,k}^{\alpha\beta}$ – unknown flow of transport blocks from i into j, passing in an arc $p_{\alpha\beta} \in P_M$, obtained from the route m_k ($u_{ij,k}^{\alpha\beta}$ determine arc flows in transport blocks on the route network G_M); $u_{ij,k}^{\eta\xi}$ – unknown flow of transport blocks from i into j, passing in an arc $p_{\eta\xi} \in P$ on the route m_k.

Required to minimize function

$$\sum_{k=1}^{l} C_{tr}^{k}\left(\left(\sum_{\eta\xi\in q_k}\sum_{ij\in S} u_{ij,k}^{\eta\xi}\right), d_k\right) + \sum_{\beta=1}^{n} C_{load}^{\beta}\left(\sum_{\alpha=1}^{n}\sum_{k=1}^{l}\sum_{ij\in S}(u_{ij,k}^{\alpha\beta} + u_{ij,k}^{\beta\alpha})\right) \quad (7)$$

subject to

$$\sum_{ij\in S} u_{ij,k}^{\eta\xi} \le W_{\eta\xi}^{k} \text{ for all } \eta\xi \in q_k,\ \ k = \overline{1,l}; \quad (8)$$

$$\sum_{\beta=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{l} u_{ij,k}^{\alpha\beta} - \sum_{\beta=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{l} u_{ij,k}^{\beta\alpha} = \begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} \text{ at } i=\alpha, \\ 0 \text{ at } i\ne\alpha, j\ne\alpha, \\ -\sum_{i=1}^{n} \tilde{a}_{ij} \text{ at } j=\alpha, \text{ for } \alpha=\overline{1,n},\ j=\overline{1,n} \end{cases} \quad (9)$$

$$\sum_{\alpha=1}^{n}\sum_{k=1}^{l}\sum_{ij\in S}(u_{ij,k}^{\alpha\beta} + u_{ij,k}^{\beta\alpha}) - \sum_{j=1}^{n}(\tilde{a}_{\beta j} + \tilde{a}_{j\beta}) \le 2b_\beta,\ \ \beta = \overline{1,n}; \quad (10)$$

$$\sum_{\beta=1}^{n}\sum_{ij\in S}(u_{ij,k}^{\alpha\beta} + u_{ij,k}^{\beta\alpha}) \le b_{\alpha}^{k},\ \ \alpha\in\upsilon_k,\ \ k=\overline{1,l}; \quad (11)$$

in specific cases of solving the problem, restrictions on the prohibition of branching flows can be added to the specified constraints

$$u_{ij,k}^{\alpha\beta} = \begin{cases} \tilde{a}_{ij}, \text{ if the flow passes in an arc } \alpha\beta\in m_k, \\ 0 \text{ otherwise,} \end{cases} \quad (12)$$

$$u_{ij,k}^{\alpha\beta} \ge 0,\ u_{ij,k}^{\eta\xi} \ge 0, \text{ and integers,} \quad (13)$$

constraints on the time of delivery of correspondence flows to the recipient are taken into account

$$t_{ij} \le \tilde{t}_{ij},\ \ ij\in S. \quad (14)$$

It is assumed that there is an operator $\Phi : u_{ij,k}^{\alpha\beta} \Rightarrow \{u_{ij,k}^{\eta\xi}\}$, $p_{\alpha\beta}\in P_M$, $p_{\eta\xi}\in P$, $ij\in S$, $k=\overline{1,l}$, which displays the flow along the route arc of the route m_k on the network G_M to the corresponding subset of the route arcs m_k on the network G.

In formulas (7)-(14) the following notations are introduced: C_{tr}^{k} is a nonlinear function that determines the dependence of transport costs on the number of transport blocks transmitted along the route m_k and the length of the route d_k. It is assumed that the present cost functions C_{tr}^{k} specified for each route with $\{m_k\}$. Transport networks are characterized by the dependence of such functions on the working fleet and carrying capacity of vehicles on the route, and data transmission networks – on the bandwidth and length of communication channels. It should be noted that the functions of the unit cost of transportation and handling of a unit of flow (one transport unit) cannot be applied here; q_k – ordered set of arcs with P, that make up the route m_k; C_{load}^{β} – nonlinear cost function for processing transport blocks in a node β; b_β, $\beta=\overline{1,n}$ – maximum throughput β-th node in transport blocks, the throughput is set for transit flows, since the outgoing and incoming flows for each node must be handled unconditionally. For nodes of the third type $b_\beta = 0$; $W^k_{\eta\xi}$ – the carrying capacity of the vehicle or the capacity of the communication channel on the route m_k on the arc $\eta\xi \in P$ in transport blocks, $W^k_{\eta\xi} \in \{w_1, w_2, ..., w_v\}$, where w_1, w_2,..., w_v – integers ordered in ascending order of positive numbers; b^k_α – limit on the maximum total number of transport blocks that can be processed at a transit node α on the route m_k; υ_k – an ordered set of nodes with N on the route m_k; t_{ij}, $\tilde{t}_{ij}$, $ij \in S$, – estimated and specified time for the delivery of correspondence flows a_{ij} with i into j.

The first component of the function determines the transport costs, the second – the costs of processing the transport blocks. Conditions (9) ensure continuity of flow, and conditions (10), (8), (9) are, respectively, constraints on the capacity of nodes; route capacity; volumes of processing of transport blocks in the nodes of vehicles or in the switched nodes of the data transmission network.

When solving a problem (7)-(14) should be obtained the final technical and economic indicators of network functioning. The results of the solution contain for each node of the network a scheme of sorting of outgoing flows with estimates of the time of delivery of correspondence to recipients, the costs of sorting flows, the costs of loading and unloading outgoing, incoming and transit transport blocks, as well as the costs of transporting (transmitting) flows of transport blocks on optimized routes and complete information about the routes of their transportation.

Due to the complexity of solving the nonlinear discrete problem (7)-(14), a method for reducing it to a set of multidimensional knapsack problems with binding constraints was proposed in [13]. The essence of the decomposition is the following. It is intuitively clear that it is possible to achieve a decrease in the value of the function (7) with a maximum increase in the values of $u^{\eta\xi}_{ij,k}$ on all arcs (sections) $\eta\xi \in P$ routes $\{m_k\}$ when distributing all flows in the network G_M by the criterion: minimum transit overloads, if the number of transit overloads is equal – minimum path length. As a result, we construct some analogue of the dual problem to the problem (7)-(13) with the exclusion of nonlinear cost functions from the explicit consideration C^k_{tr} and C^β_{load}. First, let's assume that all flows $\tilde{a}_{ij}$ distributed in the network G_M according to the specified criterion. We using a reference matrix of the distribution of flows along the shortest paths $C = \| c_{ij} \|_{n \times n}$ [29] with elements

$$c_{ij} = \begin{cases} \mu, \text{ if flow } \tilde{a}_{ij} \text{ transmitted from node } i \text{ to node} \\ j \text{ with overload at node } \mu, \\ i, \text{ if flow } \tilde{a}_{ij} \text{ transmitted from node } i \text{ to node} \\ j \text{ without overload,} \\ 0, \text{ if } i = j, \end{cases}$$

and a given set $\{m_k\}$, $k = \overline{1,l}$, for each route m_k it's not hard to find subsets $\{u^{\alpha\beta}_{ij,k}\}$ flows that could potentially be allocated to this route. Let's denote $\left| \{u^{\alpha\beta}_{ij,k}\} \right| = \varepsilon_k$.

Because G_M – multi-network, for any m_{k_1}, m_{k_2}, $k_1 \neq k_2$ and fixed $\alpha\beta$ and ij can be performed $\{u_{ij,k_1}^{\alpha\beta}\} \cap \{u_{ij,k_2}^{\alpha\beta}\} \neq \varnothing$.

Renumbering, starting with 1, non-zero flows $\tilde{a}_{ij}$ in the matrix $\tilde{A} = \| \tilde{a}_{ij} \|_{n\times n}$ in rows from left to right and in columns from top to bottom. Let's build a matrix $H = \| h_{ij} \|_{n\times n}$, in which

$$h_{ij} = \begin{cases} \text{number } h \text{ of flow } \tilde{a}_{ij}, \text{ if } \tilde{a}_{ij} \neq 0, \\ 0 \text{ otherwise, } h = \overline{1, h_{\max}}, \end{cases}$$

where $h_{\max}$ – number of non-zero elements in the matrix $\tilde{A}$, $h_{\max} \leq n^2 - n$.

Let $\xi^k = \| \xi_i^k \|_{\eta_k}$, $\eta_k = | \upsilon_k |$, $k = \overline{1,l}$ – vectors containing host numbers (G or G_M) on the routes m_k when passing them from left to right. This means that direct and return routes may or may not coincide. Let's write down the display operator of any flow $u_{ij,k}^{\alpha\beta} \in \{u_{ij,k}^{\alpha\beta}\}$ along the route arc $\alpha\beta$ upon m_k on the set of arcs of this route in the network G in the form of $\Phi : u_{ij,k}^{\alpha\beta} \Rightarrow \{u_{ij,k}^{\xi_\mu \xi_{\mu+1}}, u_{ij,k}^{\xi_{\mu+1}\xi_{\mu+2}}, \ldots, u_{ij,k}^{\xi_{\mu+c-1}\xi_{\mu+c}}\}$, where $\xi_\mu = \alpha$, μ – node sequence number α in vector ξ^k, $\mu \in [\overline{1, \eta_k})$; $\xi_{\mu+c} = \beta$, $\mu + c$ – node sequence number β in vector ξ^k, $\mu + c \in (\overline{1, \eta_k}]$.

Let's build vectors $f^k = \| f_j^k \|_{\varepsilon_k}$ and matrices $\dot{A}^k = \| \dot{a}_{ij}^k \|_{(\eta_k - 1)\times\varepsilon_k}$, $\ddot{A}^k = \| \ddot{a}_{ij}^k \|_{\eta_k\times\varepsilon_k}$, $\dddot{A}^k = \| \dddot{a}_{ij}^k \|_{\eta_k\times\varepsilon_k}$, $\ddddot{A}^k = \| \ddddot{a}_{ij}^k \|_{\eta_k\times\varepsilon_k}$, $k = \overline{1,l}$ so:

$f_j^k = h$, if the flow $\tilde{a}_{i'j'}$ with a number h can be distributed to a route m_k. Indexes i' and j' mean indices i and j variables $u_{ij,k}^{\alpha\beta} \in \{u_{ij,k}^{\alpha\beta}\}$, where $i, j = \overline{1,n}$, therefore $\tilde{a}_{i'j'}$ define the elements $\tilde{a}_{ij}$ matrix $\tilde{A}$. For convenience, plural flows $\{u_{ij,k}^{\alpha\beta}\}$ can be ordered according to increasing flows numbers $\tilde{a}_{i'j'}$ in the matrix H;

$$\dot{a}_{ij}^k = \begin{cases} \tilde{a}_{i'j'} \text{ for } i = \overline{\mu, \mu + c - 1}, \\ 0 \text{ for } i \notin [\mu, \mu + c - 1],\ i = \overline{1, \eta_k - 1}; \end{cases}$$

$$\ddot{a}_{ij}^k = \begin{cases} \tilde{a}_{i'j'}, \text{ if } (i = \mu) \vee (i = \mu + c), \\ 0 \text{ otherwise, } i = \overline{1, \eta_k}; \end{cases}$$

$$\dddot{a}_{ij}^k = \begin{cases} \tilde{a}_{i'j'}, \text{ if } (i = \mu \wedge i' \neq \alpha) \vee (i = (\mu + c) \wedge j' \neq \beta), \\ 0 \text{ otherwise, } i = \overline{1, \eta_k}; \end{cases}$$

$$\ddddot{a}_{ij}^k = \begin{cases} \tilde{a}_{i'j'}, \text{ if } (i = \mu \wedge i' = \alpha) \vee (i = (\mu + c) \wedge j' = \beta), \\ 0 \text{ otherwise, } i = \overline{1, \eta_k}, \end{cases}$$

for $j = \overline{1, \varepsilon_k}$, $k = \overline{1,l}$, where $\vee$ $\wedge$ — logical "or" and "and" signs.

Here is the algorithm for generating data.

Algorithm 1. Formation f^k, $\dot{A}^k$, $\ddot{A}^k$, $\dddot{A}^k$, $\ddddot{A}^k$

1. For $\{ k \mid k = \overline{1,l} \}$ do step 2-14.
2. $\dot{A}^k \leftarrow 0$, $\ddot{A}^k \leftarrow 0$, $\dddot{A}^k \leftarrow 0$, $\ddddot{A}^k \leftarrow 0$.
3. For $\{ j \mid j = \overline{1,\varepsilon_k}, j \equiv u^{\alpha\beta}_{i'j',k} \}$ do step 4-13.
4. $f^k_j \leftarrow h_{i'j'}$.
5. For $\{ \mu \mid \mu = \overline{1,\eta_k - 1} \}$ while $\xi^k_\mu \neq \alpha$ go to step 5.
6. $\ddot{a}^k_{\mu j} \leftarrow \tilde{a}_{i'j'}$.
7. If $i' \neq \alpha$, then $\dddot{a}^k_{\mu j} \leftarrow \tilde{a}_{i'j'}$.
8. If $i' = \alpha$, then $\ddddot{a}^k_{\mu j} \leftarrow \tilde{a}_{i'j'}$.
9. For $\{ i \mid i = \overline{\mu, \eta_k} \}$ while $\xi^k_i \neq \beta$ do $\dot{a}^k_{ij} \leftarrow \tilde{a}_{i'j'}$, go to step 9.
10. $\ddot{a}^k_{ij} \leftarrow \tilde{a}_{i'j'}$.
11. If $j' \neq \beta$, then $\dddot{a}^k_{ij} \leftarrow \tilde{a}_{i'j'}$.
12. If $j' = \beta$, then $\ddddot{a}^k_{ij} \leftarrow \tilde{a}_{i'j'}$.
13. Go to step 3. ! End of cycle by j
14. Go to step 1. ! End of cycle by k
15. End of algorithm.

Let $W^k = \| w^k_i \|_{\eta_k - 1}$, $B^k = \| b^k_i \|_{\eta_k}$, $B = \| b_i \|_n$, $k = \overline{1,l}$ – respectively, the vectors of restrictions on the carrying capacity of the vehicle or the capacity of the channel on the arcs of the route m_k; the total number of transport blocks that can be processed at the transit hub i on the route m_k; capacity of network nodes for processing transit transport blocks.

Let's introduce variables x^k_j, $j = \overline{1,\varepsilon_k}$, $k = \overline{1,l}$, where

$$x^k_j = \begin{cases} \text{one of the meanings } 1/\tilde{a}_{i'j'}, 2/\tilde{a}_{i'j'}, \ldots, \tilde{a}_{i'j'}/\tilde{a}_{i'j'}, \text{ if flow } \tilde{a}_{i'j'} \\ \text{with a number } j \text{ is selected for a route } m_k, \\ 0 \text{ otherwise.} \end{cases}$$

Taking into account the above limitation of formulas (8), (11) we write in the form

$$\sum_{j=1}^{\varepsilon_k} \dot{a}^k_{ij} x^k_{f^k_j} \leq w^k_i, \quad i = \overline{1,\eta_k - 1}, \ k = \overline{1,l}, \tag{15}$$

$$\sum_{j=1}^{\varepsilon_k} \ddot{a}^k_{ij} x^k_{f^k_j} \leq b^k_i, \quad i = \overline{1,\eta_k}, \quad k = \overline{1,l}. \tag{16}$$

Let's denote $\sum_{j=1}^{\varepsilon_k} \ddot{a}_{ij}^k x_{f_j^k}^k = \psi_{\xi_i^k}^k$, $i = \overline{1,\eta_k}$, $k = \overline{1,l}$, $\sum_{j=1}^{\varepsilon_k} \dddot{a}_{ij}^k x_{f_j^k}^k = \lambda_{\xi_i^k}^k$, $i = \overline{1,\eta_k}$, $k = \overline{1,l}$.

Then the constraints (9) and (10) are written as follows:

$$\sum_{k=1}^{l} \lambda_i^k = \sum_{j=1}^{n} (\tilde{a}_{ij} + \tilde{a}_{ji}), \quad i = \overline{1,n}, \tag{17}$$

$$\frac{1}{2}\sum_{k=1}^{l} \psi_i^k \le b_i, \quad i = \overline{1,n}. \tag{18}$$

The constraint on the prohibition of thread branching (12) can be written as

$$x_j^k = \begin{cases} 1, \text{ if a flow } \tilde{a}_{i'j'} \text{ with a number } j \text{ is selected for a route } m_k, \\ 0 \text{ otherwise.} \end{cases} \tag{19}$$

Let's build the objective function of the problem: it is required to minimize the number of l the "cheapest" busiest routes from the set $\{m_k\}$ at the maximum load of arcs (sections) of these routes. "Cheap" routes are those routes that have the lowest average annual present costs for transportation and processing of flows. It is assumed that the present cost functions C_{tr}^k specified for each route with $\{m_k\}$. Transport networks are characterized by the dependence of such functions on the operating fleet and carrying capacity of vehicles on the route, and data transmission networks – on the bandwidth and length of communication channels. Therefore, for real networks, the reduced costs will be the same both for the most loaded route and for a poorly loaded or not loaded route at all. It should be noted that the functions of the unit cost of transportation and handling of a unit of flow (one transport block) cannot be used here, since it is less for routes with higher capacity, which will inevitably lead to the loading of "more expensive" routes.

In addition, it should be borne in mind that the upper limit on the number of routes l plural $\{m_k\}$ when solving the problem, it is not fixed – after solving the problem, it may turn out to be larger than the specified one, if in the process of solving it is allowed to generate and introduce new routes if it is impossible to distribute all flows.

Let's write down the objective function

$$\min_{|\{m_k\}|, C_{tr}^k} \left\{ \{m_k\} \;\middle|\; \sum_{j=1}^{\varepsilon_k} \dot{a}_{ij}^k x_{f_j^k}^k \Rightarrow \max, \ i = \overline{1,\eta_k - 1}, k = \overline{1,l} \right\}. \tag{20}$$

Thus, as a result of the transformation of the problem (7)-(14), its solution was reduced to a certain sequence of simultaneous solution l- multidimensional knapsack problems (20) with block constraints (15), (16) dimension $(\eta_k - 1) \times \varepsilon_k$ and $\eta_k \times \varepsilon_k$, binding constraints (17), (18) and constraints (19), and (14). The need for repeated (iterative) solution of the problem is due to the fact that in one iteration all the flows specified by the matrix $\tilde{A}$, may not be distributed along the shortest paths due to violation of any restrictions. In this case, new paths different from the original ones are chosen for undistributed flows, and new subsets are defined $\{u_{ij,k}^{\alpha\beta}\}$

and again, the knapsack problem is solved, taking into account the remaining reserve of restrictions. The iterative process continues until all flows are distributed across the network. If it is not possible to distribute all flows due to violation of individual constraints, then, at the request of the designer, relaxations can be introduced interactively for all or some constraints.

The obtained combinatorial problem is also NP-hard [30-32], therefore, for its solution in different variants and modes, a number of algorithms have been proposed, which significantly use the specifics of the problem structure, abstract data types and techniques characteristic of heuristic algorithms for solving a multidimensional backpack problem. Algorithms make it possible to obtain rational, from the point of view of the network designer, solution to the problem in a reasonable time.

4. Programs Optimization of Network Structure and of Flows Routing. Numerical Experiment Results

In fact, modeling the hierarchical structure and flows routing in a communication network is a computer technology consisting of scenarios of actions of the designer and the software system when choosing input data, parameters of programs and the network to be designed. In the information and analytical decision support system (IA DSS) [28, 33], scenarios are implemented in the form of a multi-window and multi-layer graphical interface, which allows viewing the network structure and its individual fragments in a cartographic form; outgoing and incoming flows in network nodes; the values of all specified constraints and optimization parameters; optimization results for different options for solving the problem, etc. The designer in the dialog mode can change the values of the initial data and parameters of the problem, get a set of solutions and choose the most favorable of them. At the same time, he can always compare solutions for assessing the technical and economic indicators of the network, depending on the selected parameters and preference criteria. For an experimental study of solving the problems as well as for training dispatchers to work with the programs, their demo version was developed [16, 17, 34, 35]. Such a programs are included in the IA DSS and can work offline, when all the necessary input data is generated by a pseudorandom number sensor. IA DSS is being developed at the Institute of Telecommunications and Global Information Space of the National Academy of Sciences of Ukraine and in the future will function on a real-time scale and allow to effectively manage nonlinear and non-stationary processes of processing and distribution of flows at all levels of the hierarchical network.

In fig. 2 shows the main forms of the programs into which the input data is entered. In them, you can select different options for the programs and specify whether you need to optimize the network structure in an automated mode or enter the types of nodes and their service areas manually, use cost functions in the algorithms for solving the problem or not. To change the initial value of flows in the process of solving the problems, the flow prediction coefficient can be used (K_p), the initial value of which is set to one by default. At the bottom of the main forms there is a window for displaying current programs messages, and buttons for activating actions. All output data is output to data sets, and displayed on a computer screen using the system program WordPad. In the output form displays the selected network structure and the main preliminary technical and economic indicators of its functioning, and for each node of the network, an edited scheme for sorting correspondence flows and forming flows of transport blocks. If the problem of choosing a structure solve for a transport network, then at the beginning of the form, the results of solving the problem of balancing the matrix of container flows [26], which arises due to a violation of the balance conditions – the equality of the sum of outgoing and incoming containers in individual network nodes. After solving the problem of distribution and routing of flows of transport blocks, in an additional output form are displayed sorting schemes flow with final estimates of the time of their delivery to recipients, and the results of solving the routing problem, which contain final technical and economic indicators of network functioning and complete information about the transport of flows along optimized routes.

Numerical example of designing a network. Let's consider the results of mathematical modeling of solving problems of current planning on the road transport network. To conduct the experiment, a container transport network with a number of nodes was generated by a pseudorandom number sensor n =120 and the degree of nodes val =5. Arc lengths d_{kl}, $kl \in P$ were generated in the range from 80 to 300 km, and the value of the outgoing flows of cargo (goods) from the nodes of the first type was set in the range from 1 to 9 units, from the nodes of the second and third types – in the range from 1 to 5 units. It was assumed that the routes $\{m_k\}$, $k = \overline{1,l}$ vehicles coincide with the arcs of the network and traffic on the routes is allowed in both directions, so the l = 120x5/2 = 300 projected routes.

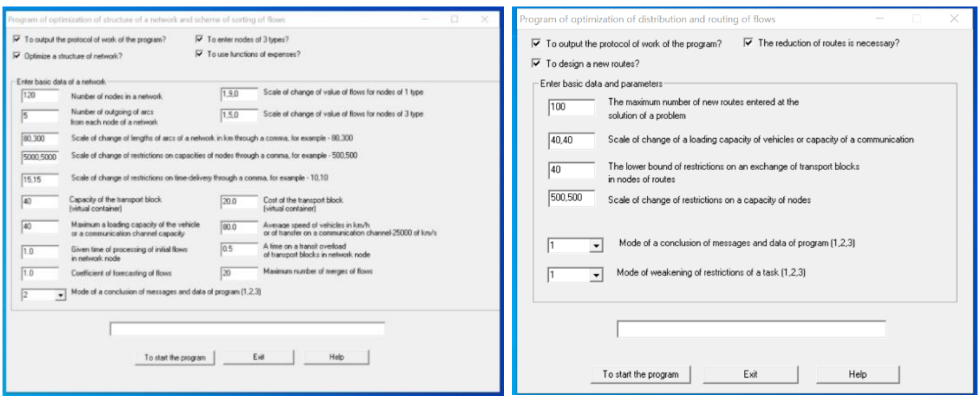


Fig. 2. Main forms of the programs

The following parameter values were taken (see Fig. 2): container size ω = 40 units of discrete cargo; time for processing (sorting) discrete cargoes in network nodes T_a =24h; carrying capacity of motor vehicles $W^k_{\eta\xi}$ on all arcs $\eta\xi \in q_k$ of all routes $\{m_k\}$, $k = \overline{1,l}$ was accepted as the same, equal W, and varied within $W \in$ {40, 20, 10} containers; time for transit transshipment of containers in network nodes T_b =12h; time of parking of vehicles at the end nodes of routes T_{end} =22h; periodicity of vehicle traffic T_{move} =24h; average speed of vehicles V_{av} = 80 km/h; daily throughput capacities of transit processing nodes h_i and b_i, $i = \overline{1,n}$ 5000 units of discrete cargo and 500 containers, respectively, were accepted; time of delivery of cargo to the recipient t_{ij} was calculated in a day, so the maximum delivery time was set in a day and accepted T_{ij}=15 days for everyone $ij \in S$; maximum number of discrete cargo bundles $\nu_{\max}$ was not restricted; branching of container flows is prohibited.

Cost functions can be either piecewise convex (gully) or convex or concave for different communication networks. To calculate the average annual present costs for transportation and processing of flows, specific functions in conventional units of cost, typical for transport enterprises, were used:

$$C_{sort}^{i}(\cdot) = c_1 x_i \exp\left(-\frac{c_2 x_i}{1+q_i}\right),\ i=\overline{1,n},$$

where $C_{sort}^{i}(\cdot)$, x_i, q_i – respectively, the costs of sorting cargo flows, the total flow, the number of directions for sorting flows to other network nodes in the node i;

$$C_{load}^{i}(\cdot) = \sqrt{c_3 u_i^2 + c_4 u_i},\ i=\overline{1,n},$$

where u_i – the sum of outbound, inbound, and transit containers handled at the node i, (in the problem of optimizing the network structure, only incoming and outgoing flows are taken into account in the cost of handling containers in network nodes, but transit flows are not taken into account);

$$C_{tr}^{ij}(\cdot) = \frac{u_{ij}}{W}\left[\frac{c_5(T_{end} + 2d_{ij}/V_{av})}{T_{move}} + c_6 + c_7 d_{ij}\right]\ \forall\ ij \in S;$$

$$C_{tr}^{k}(\cdot) = D_{ts}^{k} W(c_8 + c_9 d_k) + c_{10} D_{mb},\ k=\overline{1,l},$$

where $c_1 - c_{10}$ – задані коефіцієнти.

Here: $D_{ts}^{k} = (2T_{end} + 2d_k / V_{av}) / T_{move}$ – working fleet of vehicles on the route m_k;

$D_{mb} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij} \left\lceil t_{ij}^{c} / T_{move} \right\rceil$ – working fleet of containers in the transportation network;

$t_{ij}^{c} = 2t_a + d_{ij}^{k} / (V_{av}\theta) + \psi_{ij} t_b$ – total time of loading and unloading of containers $\tilde{a}_{ij}$ in the initial i and end j points of destination and the time of their transportation from i to j on a path of length d_{ij}^{k}, where ψ_{ij} – number of transit overloads $\tilde{a}_{ij}$ on the way from i to j, θ – time rationing coefficient (for transport networks θ= 24 hours, for data networks θ = 1).

To calculate the time of delivery of goods to the final recipient, the following formulas were used:

$$t_{ij} = \begin{cases} 2t_a + d_{ij}/(V_{av}\theta) + \psi_{ij} t_b, & \text{if } c_{ij} = i, \\ t_a(\nu_{ij} + 2) + \sum\limits_{\xi\eta \in \Omega_{ij}} (d_{\xi\eta}/(V_{av}\theta) + \psi_{\xi\eta} t_b), & \text{if } c_{ij} \neq i, \end{cases}$$

where ψ_{ij}, $\psi_{\xi\eta}$ – the number of transit transshipments of the container in which the cargo is located a_{ij} on the segment (i, j) or

$(\xi,\eta)\in\Omega_{ij}=\{(i,k_1),(k_1,k_2),...,(k_m,j)\}$, d_{ij}, $d_{\xi\eta}$ – the distance of the segment, c_{ij} – elements of the reference matrix of merging correspondence flows $C=\| c_{ij} \|_{n\times n}$ [27].

When solving the problems, we calculated costs in conventional units of cost (c.u.) after optimization: costs for sorting cargo C_{sort}, costs of loading and unloading containers into vehicles C_{load}, transportation costs C_{tr}, the total network costs $C_{all}=C_{sort}+C_{load}+C_{tr}$; average container load factor $K_{con}=\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\delta_i}\sum_{j=1}^{n}x_{ij}\,/\left(\left\lceil\frac{x_{ij}}{\omega}\right\rceil\omega\right)$, $\delta_i=\sum_{j=1}^{n}\delta_{ij}$; average number of cargo sorting directions in the network node $N_{sort}=\frac{1}{n}\sum_{i=1}^{n}\delta_i$.

Initially, 28 nodes of the second type and 92 nodes of the first type were generated for the network. The task of optimizing the network structure was solved for $W\in$ {40, 20, 10} with constant values of discrete cargo flows and other specified parameters. For all values W the same network structure was obtained, containing 79 nodes of the first type and 41 nodes of the second type. Therefore, to solve the problems of distribution and routing of flows of empty and loaded containers for $W\in$ {40, 20, 10} as input data, the results of solving the problem of optimizing the network structure, obtained at the W = 40.

Results of solving problems on the initial and optimized network for the carrying capacity of vehicles W = 40, 20, 10 are given in Table 1, 2 and in Fig. 3, 4. The following notations are adopted in the tables: C_{all}, C_{tr}, C_{sort}, C_{load} – the total network costs (C_{all} = $C_{tr}+C_{sort}+C_{load}$), costs of transport, sorting, loading and unloading containers in mln. conventional units; l, l^+, l^0 – the number of specified, entered and loaded vehicle routes; D_{tr} – working fleet of vehicles; K_{tr} – average vehicle load factor; D_{con} – working fleet of containers; K_{con} – average container load factor; N_{sort} – the average number of backbone directions for sorting discrete cargoes in the network node; $t_{\min}$, $t_{\max}$, t_{mean} – minimum, maximum and average time of delivery of discrete cargo to the consignee per day; V_1, V_2, V_3 – options for solving the problem.

In the variants V_1 and V_2 the distribution of container flows was carried out along specified routes coinciding with the arcs of the network, and if it was impossible to distribute all flows, the introduction of new routes was allowed. In the variant V_1 constraints on the time of delivery of cargo to the recipient $\tilde{T}=\| \tilde{t}_{ij} \|_{n\times n}$ were taken into account, and in the variant V_2 were weakened. In the variant V_3 for each pair of nodes (i,j) $i=\overline{1,n-1}, j=\overline{i+1,n}$ the route of the vehicle was generated in the network according to the criterion: minimum transit nodes, if the number of transit nodes is equal – minimum length. Total generated $l = n(n-1)/2$ = 120x119/2 = 7140 routes, the movement of vehicles on the routes was allowed in both directions. In the variant V_3 any flow of containers can be delivered to the recipient without transit overload. From the tables, you can see that for the V_1 and V_2 at $W = 40$ and $W = 20$ full costs with the relaxation of constraints on t_{ij} decrease slightly, and $t_{\max}$ noticeably growing (Fig. 4*a*).

Table 1

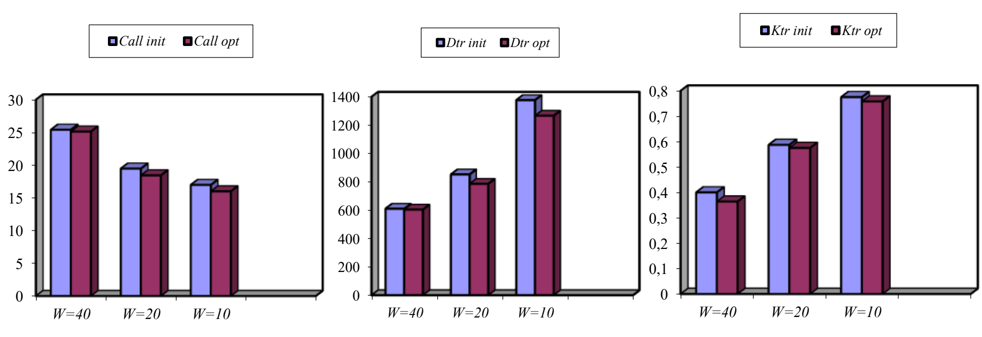
Solution Results	Initial Network $N_1 = 92$, $N_2 = 28$								
	$W = 40$			$W = 20$			$W = 10$		
	V_1	V_2	V_3	V_1	V_2	V_3	V_1	V_2	V_3
C_{all}	25,396	25,186	152,78	19,464	19,337	79,397	16,975	19,048	42,706
C_{tr}	20,002	19,770	148,17	14,069	13,488	74,791	11,581	13,030	38,100
C_{sort}	3,216	3,216	3,216	3,216	3,216	3,216	3,216	3,216	3,216
C_{load}	2,178	2,200	1,389	2,178	2,632	1,389	2,178	2,802	1,390
l	300	300	7140	300	300	7140	300	300	7140
l^{+}	5	0	0	126	97	0	388	461	0
l^{0}	305	300	900	426	397	900	688	761	900
D_{tr}	610	600	1819	852	794	1819	1376	1522	1819
K_{tr}	0,400	0,412	0,055	0,587	0,806	0,109	0,775	0,905	0,221
D_{con}	14573	14638	13041	14573	16093	13041	14573	16715	13041
K_{con}	0,637	0,637	0,637	0,637	0,637	0,637	0,637	0,637	0,637
N_{sort}	21	21	21	21	21	21	21	21	21
$t_{\min}$	2,042	2,042	2,042	2,042	2,042	2,042	2,042	2,042	2,042
$t_{\max}$	6,738	9,359	4,775	6,738	16,999	4,775	6,738	18,302	4,775
t_{mean}	4,205	4,266	3,393	4,205	5,317	3,393	4,205	5,570	3,393

At $W = 10$, when the maximum load of vehicles is reached, in the V_2 total costs are not reduced, but increased due to the distribution of flows along longer path and an increase in the working fleet of containers. Therefore, when reducing the load capacity, a better solution can be obtained given constraints on t_{ij}.

Average delivery time t_{mean} remains fairly stable for all variants (Fig. 4*b*). In the variant V_3 with a decrease in the load capacity of vehicles, there is a significant reduction in total costs at the lowest values $t_{\max}$ and t_{mean}. This means that the best solution to the problem can be obtained in the V_3 with a further reduction in load capacity and deviation of flows from the shortest paths to increase the load of vehicles.

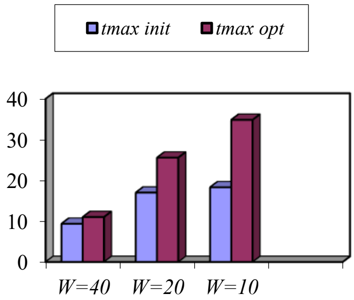
Table 2

Solution Results	Optimized Network $N_1 = 79,\ N_2 = 41$								
	$W = 40$			$W = 20$			$W = 10$		
	V_1	V_2	V_3	V_1	V_2	V_3	V_1	V_2	V_3
C_{all}	25,116	25,030	116,64	18,430	18,415	61,336	15,993	17,679	33,687
C_{tr}	19,798	19,699	111,98	13,111	12,691	56,680	10,674	11,820	29,031
C_{sort}	3,293	3,293	3,293	3,293	3,293	3,293	3,293	3,293	3,293
C_{load}	2,026	2,038	1,363	2,026	2,431	1,363	2,026	2,566	1,363
l	300	300	7140	300	300	7140	300	300	7140
l^+	2	0	0	93	71	0	334	390	0
l^0	302	300	722	393	371	722	633	690	722
D_{tr}	604	600	1455	786	742	1455	1266	1380	1455
K_{tr}	0,364	0,370	0,064	0,575	0,783	0,128	0,758	0,902	0,259
D_{con}	13945	13983	12783	13945	15329	12783	13945	15762	12783
K_{con}	0,643	0,643	0,643	0,643	0,643	0,643	0,643	0,643	0,643
N_{sort}	19	19	19	19	19	19	19	19	19
$t_{\min}$	2,042	2,042	2,042	2,042	2,042	2,042	2,042	2,042	2,042
$t_{\max}$	7,251	11,012	6,554	7,251	25,573	6,554	7,251	34,835	6,554
t_{mean}	4,275	4,311	3,488	4,275	5,367	3,488	4,275	5,594	3,488

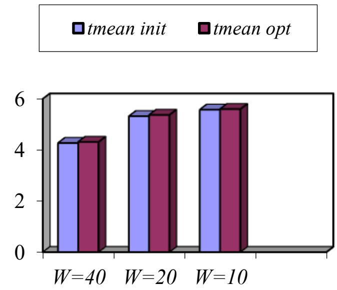


a) *b*) c)

Fig. 3. Dependence of the total average annual reduced costs (*a*), the working fleet of vehicles (*b*), the load factor of vehicles (*c*) on the carrying capacity of vehicles for the variant V_1



a)



b)

Fig. 4. Maximum (*a*) and average (*b*) time of delivery of cargo to the consignee (recipient) per day when changing the carrying capacity of vehicles for the variant V_2

Conclusion

1. The developed methodology of solving the problems of processing and distribution of discrete flows in a hierarchical multicommodity network allows selecting the hierarchical structure of the network and determining the main technical and economic indicators of its functioning when changing the initial data and parameters. The methodology can be used for current (medium-term) planning of the processes of processing and transportation of discrete correspondence flows and is aimed mainly at optimizing the use of available network resources. The methodology is based on the sequential solution of current planning problems, which are represented by a complex of mathematical models, algorithms and computer programs.

2. The developed computer technologies for solving the problems of choosing the hierarchical structure of the network, the scheme of sorting, distribution and routing of flows allow:

• in an interactive mode to simulate various variants of the network, changing the topology, hierarchical structure, routes of vehicles or information transmission, flows, parameters and constraints of the model, and from the family of results to choose the best option taking into account the selected goal function and accepted constraints;

• to increase the efficiency of the network at the level of current planning by optimizing the use of its available resources and reducing operating costs, which makes it possible to reduce tariffs for the transportation of goods or the transfer of information, attract additional customers and ensure a constant increase in profits;

• calculate preliminary technical and economic indicators of the network functioning at the specified and forecast values of flows, estimate the cost of additional resources and plan the amount of necessary investments for the modernization and development of its structural elements, which ultimately makes it possible to increase the efficiency of the network by reducing the scarce material, raw materials, energy, financial and labor resources;

• promptly redistribute flows in case of equipment failures in nodes and transport routes, unforeseen situations, natural disasters, etc.

3. The developed instrumental software tools are designed for mathematical (computer) modeling of the functioning of communication networks with discrete flows in the infrastructure complexes of large cities, regions and the country as a whole. They can be used to solve the problems of current planning and management in transport networks, backbone data transmission networks, cellular and postal communications, etc., as well as to solve the problems of operational management and long-term development and allow to increase the estimated efficiency of the functioning of the designed communication networks. The developed tools can also be used to model and optimize the functioning of traditional logistic production and transport and storage systems, including nodes of suppliers of raw materials, production of goods, warehouses and end users. In addition, the results of the work can be useful in the educational process in the training of specialists in the field of modeling and design of complex distributed multicommodity networks.

4. Experimental studies have shown high computational efficiency of the proposed algorithms and programs, and they can be recommended for the practical solution of problems of optimizing the processes of processing and transporting flows in high-dimensional communication networks (the counting time of all problems on a transport network containing 120 nodes and 300 unoriented arcs did not exceed 30 s on a PC with an Intel Core 2 Duo processor with a clock frequency of 2.66 GHz and 2 GB of RAM).

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О.М. Трофимчук, В.О. Васянін

ДВОЕТАПНА ЗАДАЧА ОПТИМІЗАЦІЇ СТРУКТУРИ ТА МАРШРУТИЗАЦІЇ ПОТОКІВ В ІЄРАРХІЧНІЙ БАГАТОПРОДУКТОВІЙ МЕРЕЖІ

Анотація. У роботі розглядається методологія математичного моделювання двоетапної задачі оптимізації магістральної ієрархічної комунікаційної мережі з багатопродуктовими дискретними потоками і параметрами. Методологія заснована на послідовному розв’язанні задачі оптимізації структури мережі і задачі розподілу і маршрутизації потоків дискретних кореспонденцій. Як правило, такі мережі складаються з децентралізованої магістральної мережі та фрагментарних мереж у внутрішніх зонах обслуговування магістральних вузлів. Визначається чотири типи вузлів мережі і три рівні її ієрархії. У багатопродуктовій мережі кожен вузол може обмінюватися кореспонденціями (продуктами, товарами, вантажами, повідомленнями) з іншими вузлами. Кореспонденція характеризується вузлом-джерелом, вузлом-стоком та величиною, яка для транспортних мереж задається кількістю тарно-штучних вантажів в упаковці уніфікованого розміру, а для мереж передачі даних – кількістю байт, кілобайт і т.п. У транспортній магістральній мережі всі кореспонденції спочатку сортуються за адресами призначення, пакуються у транспортні блоки (контейнери), а потім транспортуються у транспортних засобах по транспортних магістралях. У мережах передачі даних кореспонденції також сортуються за адресами призначення (мультиплексуються), пакуються у віртуальні транспортні блоки, а потім передаються по магістральних каналах зв'язку. Розмір (ємність, обсяг) транспортного блоку задається параметром і визначається кількістю одиниць кореспонденцій, що у нього вміщуються. Наведено математичні моделі задач оптимізації структури мережі, розподілу і маршрутизації потоків та приклад числового моделювання розв'язання задач на транспортній мережі, що містить 120 вузлів і 300 неорієнтованих дуг. Експериментальні дослідження показали високу обчислювальну ефективність запропонованих алгоритмів і програм. Вони можуть бути рекомендовані для практичного вирішення задач оптимізації процесів обробки і транспортування потоків у комунікаційних мережах великої розмірності.

Ключові слова: багатопродуктові ієрархічні мережі, дискретні потоки, задачі комбінаторної оптимізації, математичні моделі, комп'ютерне моделювання.

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Трофимчук Олександр Миколайович
член-кореспондент НАН України, доктор технічних наук, професор, директор, Інститут телекомунікацій і глобального інформаційного простору НАН України
Адреса робоча: 03186, Україна, м. Київ, Чоколівський бульвар, 13
ORCID ID: https://orcid.org/0000-0003-3358-6274 ***e-mail:*** itgis@nas.gov.ua

Васянін Володимир Олександрович
доктор технічних наук, старший науковий співробітник, завідувач відділу прикладної інформатики, Інститут телекомунікацій і глобального інформаційного простору НАН України
Адреса робоча: 03186, Україна, м. Київ, Чоколівський бульвар, 13
ORCID ID: https://orcid.org/0000-0003-4046-5243 ***e-mail:*** archukr@meta.ua