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# SEARCH FOR AN EQUAL-STRENGTH CONTOUR INSIDE A VISCOELASTIC RECTANGLE

Annotation. Irregularity of geometric and physical parameters in thin-walled structures leads to significant concentrations of stresses and creates dangerous zones for the spread of cracks or plastic deformations. Under the influence of a tense state, they are similar to gills. Stress concentration zones in areas of irregularity have a significant impact on the tensile strength and durability of thin-walled structures. Traditional analytical and numerical methods known at this time are less effective in investigating the stress-strain condition of corrugated thin-walled structures. It is, therefore, necessary to develop new effective methods for solving the tasks of this class. Currently, for engineering calculations, there is virtually no comparison of simple and convenient formulas for determining the critical compressive load taking into account the peculiarities of the design. The scientific novelty of the paper is that to achieve the set goal, it will be used for the first time in the general theory developed for the calculation of buildings and structures, known as the "Theory of elasticity in ordinary differential equations." The paper will show that the accuracy of this new theory is adequate to the classical elongation theory and at the same time dramatically simplifies the solution of any problem in the calculation of tiles, which is achieved by converting them to conventional differential equations. The general methods of compiling differential equations, the methods of its simplification, for the calculation of membranes with cross-sectional incisions, and the calculation of plates under conditions of nonlinear deformation are discussed. Methods for solving differential equations with variable and momentum coefficients are specified. An algorithm and a program for the analysis of the stress-strain state of spatial structures and their elements are developed. The practical value of the paper lies in the possibility of using developed methods and programs for the design and construction of buildings, as well as for the stability tasks of slabs with holes, and panels used in construction as typical assembly elements. The given mathematical algorithm and program for specific tasks, which are distinguished by simplicity, can be used by design and research organizations in the calculation and design of plates and membranes.

*Keywords: Kelvin-Voigt model; Kolosov-Muskhelishvili formulas; Riemann-Hilbert problems; Volterra equation.* 

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### 1. Problem Statement

Let S be a doubly connected region whose outer boundary is a rectangle A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, A<sub>4</sub>, whose sides are parallel to the coordinate axes, and whose inner boundary is a smooth closed contour (an unknown part of the boundary of the region S). It is assumed that normally compressive stresses with Known principal vectors act on the sides of the rectangle (or constant normal displacements are given  $V_n(\sigma) = \text{contst}$ ), and the inner part (the desired equal-strength contour) is free from extrenal forces. The equal strength of the desired contour lies in the fact that the tangential normal stress acting on it at each point of the contour takes the same value depending only on time t, i.e.  $\sigma_{\vartheta}(z, t) = K_0(t)$ . The viscoelasticity of the S region as understood by the Kelvin-Voight model.

To solve the problem, methods of complex analysis are used (methods of the theory of conformal mappings and boundary value problems of analytic functions), and the equation of the desired contour is written in an analytical form.

Similar problems of the plane theory of elacticity, plate bending and extended systems are considered in [G. Kapanadze (2003, 2007), S. Shavlakadze, G. Kapanadze, A. Gogolauri (2019)] and Ukrainian scientists [16-22].

## 2. Problem solution

Let us present some results from [9] and monograph [10]. In particular, the boundary conditions of the second main problem of the plane theory of viscoelasticity acording to the Kelvin-Voight model can be written in the form [R. Banrsuri (2006, 2007), R. Banrsuri, G. Kapanadze (2013), D. Gurgenisze, G. Kipiani (2020), M. Mikeladze (2018), R. Tskedadze, D. Tabatadze (2019), G. Kifiani, G. Akhalaia, V. Beridze, G. Gegenava (2012)].

$$\int_{0}^{t} \frac{ae^{*}e^{k(\tau-t)}\varphi(\sigma,\tau) +}{\left(\varphi(\sigma,\tau) - \sigma\overline{\varphi'(\sigma,\tau)} - \overline{\psi(\sigma,\tau)}\right)}.$$
(1)  

$$2\mu(u+iv), \sigma \in L$$

Or

$$\int_{0}^{t} \left[ ae^{*}e^{k(\tau-t)} + 2e^{m(\tau-t)} \right] \Phi(\sigma,\tau) d\tau -$$

$$-\int_{0}^{t} e^{m(\tau-r)} \left( \Phi(\sigma,\tau) + \overline{\Phi(\sigma,\tau)} \right) d\tau = 2\mu^{*}(u'+iv');$$

$$\Phi(\sigma,\tau) = \phi'(\sigma,t), \ \sigma \in L$$

$$(2)$$

And the first boundary condition of the main problem has the form [10]:

$$\varphi(\sigma, t) + \sigma \overline{\varphi'(\sigma, \tau)} + \overline{\psi'(\sigma, \tau)} = i \int_0^\sigma (X_n + iY_n) ds .$$
  
$$\sigma \in L$$
(3)

Or

$$\Phi(\sigma, t) + \overline{\Phi(\sigma, t)} + \sigma \overline{\Phi'(\sigma, \tau)} + + \overline{\Psi'(\sigma, \tau)} = N(\sigma, \tau) + iT(\sigma, t) .$$

$$\sigma \in L$$
(4)

Where

 $L = L_1 \cup L_0$ ;  $L_1 = U_{k=1}^4 L_k^{(1)}$ ,  $L_k^{(1)}$  – the sides of the rectangle,  $L_0$  – the boundary of the holes, and by t we will always mean the time parameter.

Taking into account (4), condition (2) can be written in the form:

$$\Gamma\Phi(\sigma,t) - M[N(\sigma,t) + iT(\sigma,t)] = 2\mu^*(u' + iv').$$
<sup>(5)</sup>

Where  $\Gamma$  and M are time operators t:

$$\Gamma\Phi(\sigma,t) = \int_0^t \left[ae^*e^{k(\tau-t)} + 2e^{m(\tau-t)}\right] \Phi(\sigma,\tau) d\tau ; \qquad (6)$$

$$M[N(\sigma,t) + iT(\sigma,t)] = \int_0^t e^{m(\tau-t)} \left[N(\sigma,t) + iT(\sigma,t)\right] d\tau .$$
<sup>(7)</sup>

Given that  $(u + iv) = (V_n + iV_\tau)e^{i\alpha(\sigma)}$ ,  $(\alpha(\sigma)$  the angle between the ox axis and the outer normal to the contour  $L_1$  at the point  $\sigma \in L_1$ ,  $V_n(\sigma) = const$ ,  $V_\tau(\sigma) = 0$ ,  $\sigma \in L_1$ ,  $V_n(\sigma) = V_\tau(\sigma) = 0$ ,  $\sigma \in L_0$ ;  $T(\sigma, t) = 0$ ,  $\sigma \in L_1$ ;

$$N(\sigma,t) = T(\sigma,t) = 0, \, \sigma \in L_0, \, \operatorname{Re} \Phi(\sigma,t) = \frac{\sigma_{\vartheta}(\sigma,t)}{4} = \frac{K_d(t)}{4}, \, \sigma \in L_0,$$

from (5) we obtain

$$Re \Gamma \Phi(\sigma, t) = \Gamma K(t), \sigma \in L_0;$$
  

$$I_m \Gamma \Phi(\sigma, t) = 0, \sigma \in L_1.$$
(8)

Where  $K(t) = \frac{K_0(t)}{4}$ .

From (8) we obtain the Riemann-Hilbert boundary value problem:

$$Re[\Gamma\Phi(\sigma,t) - \Gamma K(t)] = 0, \ \sigma \in L_0;$$

$$Im[\Gamma\Phi(\sigma,t) - \Gamma K(t)] = 0, \ \sigma \in L_1.$$
(9)

Let the function  $z = \omega(\zeta)$  conformally map the domain S onto a circular ring  $D = \{1 < |\zeta| < R\}$  and introduce the notation  $l = l_0 \cup l_1$ , where  $l_0 = \{|\zeta| = 1\}$  and  $l_1 = \{|\zeta| = R\}$  are line samples  $L_0$  and  $L_1$  under the mapping  $z = \omega(\zeta)$ .

From (9) after mapping the area S to D, we obtain the Rienmann-Hilbert boundary value problem for the circular ring D.

$$\begin{aligned} &\operatorname{Re}\left[\Gamma\Phi_{0}(\eta,t) - \Gamma K(t)\right] = 0, \quad \eta \in l_{0};\\ &\operatorname{Im}\left[\Gamma\Phi_{0}(\eta,t) - \Gamma K(t)\right] = 0, \quad \eta \in l_{1}. \end{aligned} \tag{10}$$

Where  $\Phi_0(\zeta, t) = \Phi[\omega(\zeta), t]$ .

Problem (10) has only a trivial solution, and thus to determine the function,  $\Phi_0(\zeta, t)$  we obtain

$$\Gamma\left[\Phi_0(\zeta,t) - K(t)\right] = 0 . \tag{11}$$

It is easy to show that equation (11) has only a trivial solution and, thus for the function,  $\Phi(z,t)$  we obtain the formula:

$$\Phi(z,t) = K(t), \ z = S \ . \tag{12}$$

Therefore, for the complex potential  $\varphi(z,t)$ , taking into account the equality  $\varphi'(z,t) = \phi(z,t)$  we will have:

$$\varphi(z,t) = z \cdot K(t) \,. \tag{13}$$

Taking into account the equality  $X_n + Y_n = (N + iT)e^{i\alpha(\sigma)}$ , and taking into account (13), from (1) and (3) we obtain:

$$e^{-i\alpha(\sigma)}\Gamma[\sigma K(t)] = 2\mu^* V_n(\sigma) + MC(\sigma)$$

$$\sigma \in L_1, \ \Gamma[\sigma K(t)] = 0, \ \sigma \in L_0.$$
(14)

Where 
$$C(\sigma) = i \int_0^t N(\zeta_0) e^{i[\alpha(\zeta_0 - \alpha(\zeta))]} d\zeta_0 = \sum_{j=1}^r \int_{L_1^{(j)}} N(\zeta_0) \sin[\alpha_j - \alpha_r] ds_0 = C_{r_2} = const, \ \sigma \in L_1, \ r = 1, 4.$$

Boundary condition (14) after mapping the domain S onto D differentiating along the arc abscissa, taking into account the piecewise constancy of the right side of (14), can be written as:

$$\operatorname{Re}\left[e^{-i\alpha}i\eta\Omega(\eta,t)\right] = 0, \quad \eta \in l_{1};$$

$$\operatorname{Im}\left[i\eta\Omega(\eta,t)\right] = 0, \quad \eta \in l_{0}.$$
(15)

Where

$$\Omega(\eta, t) = \Gamma[K(t)\omega'(\eta, t)].$$
<sup>(16)</sup>

Consider the function:

$$T(\zeta) = \left(1 - \frac{1}{\zeta}\right)^2 \prod_{j=1}^{\infty} \left(1 - \frac{1}{R^{2j}\zeta}\right)^2 \cdot \left(1 - \frac{\zeta}{R^{2j}}\right)^2 \tag{17}$$

It is easy to show that  $T(\zeta)$  we satisfy the condition:

$$\overline{T(\eta)} = \eta^2 T(\eta), \ \eta \in l_0; \ \overline{T(\eta)} = T(\eta). \ \eta \in l_1.$$

And consequently, the boundary conditions (15) with respect to the function  $\chi(\eta,t) = \frac{\Omega(\eta,t)}{T(\eta)}$ , can be written in the form:

$$\operatorname{Re}\left[i\eta e^{-i\alpha(\eta)}\chi(\eta,t)\right] = 0, \quad \eta \in l_{1};$$

$$\operatorname{Im}\left[i\chi(\eta,t)\right] = 0, \quad \eta \in l_{0}.$$
(18)

The solvability condition for problem (18) hass the form  $\prod_{j=1}^{4} \left(\frac{a_j}{R}\right)^{-\frac{1}{2}} = 1$ , and the lution of this class problem itself h<sub>0</sub> (for this class see [10]) is represented by the

solution of this class problem itself  $h_0$  (for this class, see [10]) is represented by the formula:

$$\chi(\zeta, t) = E(\zeta). \tag{19}$$

Where

$$E(\zeta) = K^0 \cdot \prod_{k=1}^4 \left( 1 - \frac{a_k}{\zeta} \right)^{-\frac{1}{2}} \cdot \prod_{j=1}^\infty \prod_{k=1}^4 \left( 1 - \frac{\zeta}{R^{2j} a_k} \right)^{-\frac{1}{2}} \cdot \left( 1 - \frac{a_k}{R^{2j} \zeta} \right)^{-\frac{1}{2}}.$$
 (20)

(K<sup>0</sup> - real constant)

Thus, from (16) and (19) we finally obtain:

$$\Gamma[K(t)\omega'(\eta,t)] = T(\zeta) \cdot E(\zeta) .$$
<sup>(21)</sup>

Where  $T(\zeta)$  and  $E(\zeta)$  are defined by formulas (17) and (20), respectively.

Thus, the definition of a conformally mapping function, and thus the definition of the equation of the desired equal-strength contour, is reduced to solving an equation of the Voltaire type (21).

Introducing the notation:

$$K(t)\omega'(\zeta,t) = \Omega(\zeta,t) T(\zeta)E(\zeta) = F_0(\zeta) .$$
<sup>(22)</sup>

From (6) and (21) we obtain the equation:

$$\int_{0}^{t} \left[ a^{*} e^{k(\tau-t)} + 2e^{m(\tau-t)} \right] \Omega(\zeta,t) = F_{0}(\zeta) .$$
(23)

Differentiating (23) with respect to t and adding the resulting equality with (23) multiplied by m, we get:

$$(m-k) \mathfrak{a}^{*} \int_{0}^{l} e^{k\tau} \Omega(\zeta,\tau) d\tau + (\mathfrak{a}^{*}+2) e^{kt} \Omega(\zeta,t) = m F_{0}(\zeta) e^{kt}.$$
(24)

From (24) by differentiation with respect to t we obtain a differential equation of the first kind

$$\dot{\Omega}(\zeta, t) + a\Omega(\zeta, t) = F(\zeta) .$$
<sup>(25)</sup>

Where

$$a = \frac{m a^* + 2k}{a^* + 2}; \ F(\zeta) = \frac{k m F_0(\zeta)}{a^* + 2} \ . \tag{26}$$

(in the expression  $\dot{\Omega}(\zeta, t)$  the dot means the derivative with respect to t).

Based on the consideration that in the Kelvin-Voight model, the deformations (hence, the stresses) change exponentially, in the future we will assume that the function K(t) has the form:

$$K(t) = \sigma_0 (1 + e^{-\varepsilon t}) \quad . \tag{27}$$

Where  $\sigma_0$  and  $\varepsilon$ -are positive constants.

The solution of equation (25) has the form

$$\Omega(\zeta, t) = e^{-at} \cdot \left[ \Omega(\zeta, o) + \frac{F(\zeta)}{a} \cdot (a^{at} - 1) \right]$$
(28)

From (22) and (24) we have  $\Omega(\zeta, t) = K(o)\omega'(\zeta, o) = KF(\zeta)$  and thus for the conformally mapping function, we finally obtain the formula:

$$\omega'(\zeta, t) = \frac{F(\zeta)}{a\sigma_0(1+e^{-\varepsilon t})} [1 + (ak - 1) \cdot e^{-at}].$$
<sup>(29)</sup>

Where  $\alpha$  and  $F(\zeta)$  – are defined by formula (26).

After determining  $\omega'(\zeta, t)$  the equation of the desired contour, it will be written in the form

$$\sigma_{\zeta}' = \frac{i\eta\omega'(\eta,t)}{|\omega'(\eta,t)|}, \ \sigma \in L_0, \eta \in l_0.$$

## 3. Conclusion

The condition of equal strength of the desired contour is that the tangential normal stress on it takes a constant value. Note that the mentioned voltage is a function of point and time. To solve the problem, methods of the theory of conformal mappings and boundary value problems of analytic functions are used, and the equation of the desired equal-strength contour is constructed efficiently.

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# Г. Капанадзе, В. Балавадзе, Л. Крістесіашвілі, В. Арчвадзе ПОШУК РІВНОМІЦНОГО КОНТУРУ ВСЕРЕДИНІ В'ЯЗКОПРУЖНОГО ПРЯМОКУТНИКА

Анотація. Нерівномірність геометричних і фізичних параметрів у тонкостінних конструкціях призводить до значної концентрації напружень і створює небезпечні зони поширення тріщин або пластичних деформацій. Під впливом напруженого стану тонкостінні конструкції набувають ребристого вигляду.

Зони концентрації напружень в зонах нерівностей істотно впливають на міцність на розрив і довговічність тонкостінних конструкцій. Традиційні аналітичні та чисельні методи, відомі на даний час, менш ефективні для дослідження напруженодеформованого стану гофрованих тонкостінних конструкцій. Тому необхідно розробити нові ефективні методи вирішення завдань цього класу. На сьогодні для інженерних розрахунків практично не існує порівняння простих і зручних формул для визначення критичного навантаження на стиск з урахуванням особливостей конструкції. Наукова новизна роботи полягає в тому, що для досягнення поставленої мети вона вперше буде використана в загальній теорії, розробленій для розрахунку будівель і споруд, відомій як «Теорія пружності в звичайних диференціальних рівняннях». У роботі показано, що точність цієї нової теорії адекватна класичній теорії подовження і в той же час різко спрощує вирішення будь-якої задачі при розрахунку плит, що досягається шляхом їх перетворення в звичайні диференціальні рівняння.

Розглянуто загальні методи складання диференціальних рівнянь, методи їх спрощення для розрахунку мембран з поперечними розрізами, розрахунку пластин в умовах нелінійного деформування. Уточнено методи розв'язування диференціальних рівнянь зі змінними та імпульсними коефіцієнтами. Розроблено алгоритм і програму для аналізу напружено-деформованого стану просторових конструкцій та їх елементів. Практична цінність роботи полягає в можливості використання розроблених методів і програм для проєктування і будівництва будівель, для задач стійкості плит з отворами і ребрами, панелей, що використовуються в будівництві як типові складальні елементи. Наведені математичний алгоритм і програма для конкретних завдань, що відрізняються бути використані проєктними науково-дослідними простотою, можуть та організаціями при розрахунку та проєктуванні пластин і мембран. Наукові дослідження проводилися шляхом чисельних і практичних експериментів.

**Ключові слова:** модель Кельвіна-Фойгта; формули Колосова-Мусхелішвілі; задачі Рімана-Гільберта; рівняння Вольтерра.

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