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TESTING A NUMERICALLY-ANALYTICAL METHOD FOR PREDICTION DESIGN MAXIMA DISCHARGES OF FLOODS USING PLOTTING POSITION FORMULAS: THE RIVER UZH CASE, THE “UZHHOROD” GAUGING STATION DATA

***Abstract.** There are a lot of analytical probability distributions that might be used to predict peak discharges of floods. However, there is no proper theoretical or another similar justification for choosing an appropriate parametric probability distribution to predict peak discharges of floods by using observed data. As a permissible hypothesis, any of recommended probability distributions can be considered providing it meets the given statistical criteria and other considerations for the adequacy of simulation are taken into account. In turn, more than seventeen plotting position formulas have been proposed. They provide a non-parametric means to estimate the observed data probability distribution. Using a plotting position formula, a plot of the estimated values from a theoretical parametric probability distribution can be compared with the observed data.*

The choice of a better plotting position formula for fitting the different probability distributions has been discussed many times in hydrology and statistical literature. However, no specific criterion for choosing these formulas has been proposed yet. Perhaps there is no need for such a criterion. Maybe, the diversity of estimates that can be obtained due to these formulas matters more. Due to the diversity of the different plotting position estimates, from the point of view of informational entropy, different plotting position formulas enable revealing epistemic (non-stochastic or subjective) uncertainty in predictions of hydrological extremes.

Results of calculating empirical annual probabilities of exceedance observed maxima discharge employing various plotting position formulas show that increasing the predicting horizon toward low probable and more extreme events increases the divergence between the estimates obtained using the different plotting position formulas. Therefore, it is reasonable to assume that this divergence may be extrapolated to predict design maxima discharges of floods based on empirical estimates of plotting position probabilities.

This paper proposes a numerically-analytical method using such an extrapolation. It is based on using different plotting position formulas, numerical calculations of plotting position probabilities, and extrapolation of the divergence between the obtained estimates. The method is tested in predicting the maxima discharges of 0.5% and 1% annual probability of exceedance for the Uzh River flowing in the Transcarpathia region, the hydrological station “Uzhhorod” data.

***Keywords:** Annual probability of exceedance; divergence indicator; extrapolation; floods; numerically-analytical method; plotting position formulas; probability distributions; prediction; return period.*

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1. Introduction

Among natural disasters, riverine floods are the most common in terms of frequency, area of distribution, and losses in Ukraine [1, 2]. Annually, floods in rivers challenge people in the country because of damage to the infrastructure, losses of resources, and economy, personal property, crop losses, and threats to human health and life [3, 4].

Statistics show the annual average flood losses in the country in 1995-1998 amounted to more than UAH 900 million, in 1999-2007 more than UAH 1.5 billion, and in 2008-2010 – about UAH 6 billion [4]. From the year 2000, more than 280 emergency flood events occurred in Ukraine. Specific losses per one flood reached UAH 6,203,750 or EU 228,079; expenses for liquidation adverse consequences of one flood event – UAH 65,419,925 or EU 2,405,144 [4].

Most often, disastrous floods occur in the western regions of the country. In particular, on the Carpathian rivers [3, 5], floods are considered a common natural phenomenon [6]. So, one of the most destructive floods occurred in the Ukrainian Carpathians at the end of July 2008 [7]. The flood covered areas in Ukraine, Moldova, and Romania causing 47 fatalities and the evacuation of about 40,000 people. Then, over 40,000 houses and 33,000 ha of farmland were flooded in Ukraine [4]. It should be noted [8] the Ukrainian Carpathians and the Tisza River, Dniester, Prut, and Siret basins are among the most flood-prone regions in Europe and in the world. In addition, there is a threat of an increase in flood hazards in Ukraine in the future. In particular, it is associated with global and local climate change [6, 9], which is one of the topical problems in the country [10].

River floods will continue to challenge people harmfully [3]. They are the most common among repeatedly occurring natural disasters in the world [11]. However, river flooding is a natural hazard against which precautionary measures are most effective compared with other natural hazards [11, 12]. For centuries, people have managed river flood risks using specialised infrastructures, such as dams, river dykes and levees, drainage systems, and others, including so-called nature-based solutions [12-15]. In 2007, recognizing this, the European Union (EU) implemented the EU Flood Directive (Directive 2007/60/EC [16]). This Directive alleges that “Floods are natural phenomena which cannot be prevented”, as well as that “It is feasible and desirable to reduce the risk of adverse consequences, especially for human health and life, the environment, cultural heritage, economic activity and infrastructure associated with floods”. Nowadays, there are a lot of world and European regulatory practices in flood risk management, which have enabled the reduction of flood hazards using the reliable control of floods, infrastructure protection, and mitigation of the risk of adverse consequences [17]. Numerous data confirm the efficiency of the implementation of different flood control measures and flood risk management procedures [18-21]. In particular, the share of insured flood losses has become more notable in recent years. If only 12% of losses caused by flood events in 1980-2019 were insured [11], yet in 2021 some 22% of such losses were insured [22]. It should be noted that many flood losses relate to public infrastructure – roads, railways, dykes, riverbeds, and bridges, which are usually uninsured. Moreover, even in highly-developed industrial countries, the comparatively low share of insured flood losses has been partially due to a limited range of insurance covers in some regions and low demand, including locations well-known to be at risk of flooding [22].

However, the fixed rise in the share of insured flood losses may indicate more confidence from insurance companies in the quantitative assessment of flood risks and modern flood risk management procedures [23]. As a result, the overall trend in flood losses (after adjustment for increases in values) has fallen in Europe – despite repeated severe floods, such as those in 2002 and 2013 [24]. There are also indications in North America and China that protective measures have reduced adjusted flood losses [11, 23].

Today, Ukraine is at the stage of legal approximation to the EU Flood Risk Directive [4, 5]. In particular, according to the EU-Ukraine Association Agreement, the preparation of flood risk assessment procedures and flood hazard mapping should have been done by November 2020, and the Flood Risk Management Plans – by November 2022 [25]. However, these works are still not to be completed [4, 5, 25, 26].

Admittedly, in any field of human activity, one of the critical implementation challenges of effective risk management is an information problem [27]. More uncertainty is more risk. Savage (1954) argued that all uncertainties can be reduced to risk, converting risk assessment to the assessment of probabilities [28]. Therefore, the quantitative flood risk assessment will require the analysis and quantitative assessment of the probabilities (frequencies) of adverse or disastrous floods. Returning to the problem of flood risk management, it should be reminded that Directive 2007/60/EC [16] defines flood risk quantitatively as “the combination of the probability of a flood event and of the potential adverse consequences for human health, the environment, cultural heritage and economic activity associated with a flood event”.

In practice, according to Directive 2007/60/EC [16], the flood risk management projects' development and implementation requires estimating predicted flood water levels h_p corresponding to certain design annual probabilities of exceedance P (year^{-1}) or return periods $T_{r,p} = P^{-1}$ (years). For example, the design annual exceedance probabilities in terms of prediction of maximum water levels and possible inundation zones because of floods may be established at 0.005, year^{-1} (or 0.5%, year^{-1}), 1%, 2%, 5%, and 10%, or something else; the corresponding return periods of the design floods – 200, 100, 50, 20, and 10 years, etc. In hydrological investigations relating to river floods, these estimations are usually done by statistically analysing the frequency of flood peak discharges [29-33]. Practically, it is done in such a way. Direct annual maximum water levels' h (m) measurements at a near-located gauging (hydrological) station are converted into maxima annual discharges Q (m^3/s) by using a rating curve $Q = f(h)$ [7, 33]. As a result of long-term (not less than 30-40 years) uninterrupted hydrological measurements, time series of annual maximum water discharges of floods are formed. Gathered data are statistically analysed, and, in the frame of the stationary hypothesis, a relevant maxima annual discharges' probability distribution is chosen, which has to fit the observed data [29, 34, 35]. This probability distribution is used to derive a predicted peak discharge corresponding to a chosen design return period $T_{r,p}$ or a chosen design annual probability of exceedance P [33]. In the next step, the established peak discharge of a chosen design annual probability of exceedance may be used as the input value for the hydraulic modelling to derive the corresponding design flood level [33, 36], taking into account the current conditions with hydromorphological

characteristics of the river channel and floodplain [37]. As a result, possible inundation zones because of floods with different annual probabilities of exceedance (return periods) may be identified. This will allow providing measures to prevent and/or mitigate the flood hazard in flood-prone locations, to build or retrofit dams, dikes, levees, polders, and other hydraulic works, to assess the possible adverse consequences of flooding due to different floods, to carry out flood hazard mapping, and to assess the risks of damages as combinations of the probabilities of different flood events and the potential adverse consequences associated with these floods.

2. What can be wrong with the framework of this study?

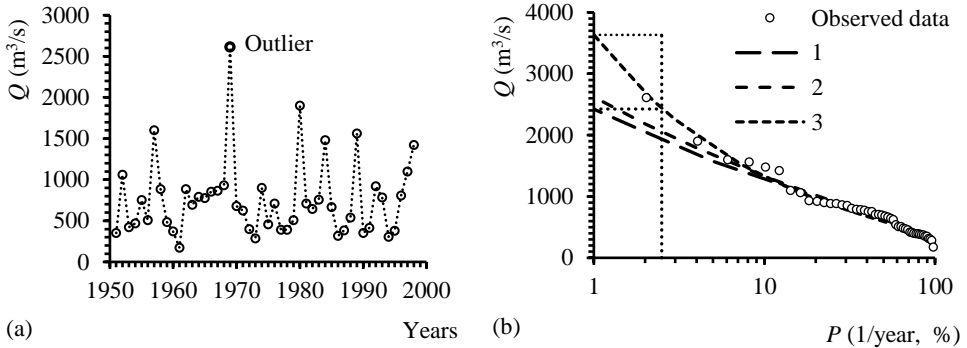
Many unsolved problems in hydrology can impact flood risk management policy [38]. One of these is the problem of how to take into account the non-stationary in hydrological predictions [39-42]. In current flood risk management projects, the observed hydrological data are considered and analysed in the frame of the stationary hypothesis. To take into account in the probabilistic modelling of flood frequency the non-stationary in hydrology, only some trivial corrections are usually proposed for the standard probability distributions. For example, this is the incorporation of trends in the parameters of the distributions, the incorporation of trends in statistical moments, or using the quantile regression method and the local likelihood method [40-42].

However, among different challenges and issues that can complicate the quantification of flood risks, there is a problem of recognising and overcoming two basic kinds of information uncertainty relating to hydrological predicting in the frame of the conventional stationary hypothesis in hydrology and water management: natural (stochastic) uncertainty and epistemic (non-stochastic or subjective) uncertainty [43]. It is quite possible that the increasing effort to develop and apply non-stationary models in hydrologic frequency analyses under changing environmental conditions can be frustrated if the additional uncertainty related to the non-stationary model complexity is accompanied by the sampling information uncertainty [44].

The natural or stochastic uncertainty stems from the essential variability of the river runoff stochastic process [43]. Available data can be insufficient to define the risk of extreme events more precisely [28]. Hydrological maxima are specific extreme events. In theory, they are not limited to the upper limit. Usually, time series of observed pick discharges hold an essential positive asymmetry (skewness); sometimes – strong outliers [29]. Often, expanding the observation periods increases the time series asymmetry (skewness) [29, 45]. That can complicate the choice of a relevant theoretical probability distribution.

Epistemic or non-stochastic uncertainty results from incomplete knowledge about the river runoff phenomena. Hydrologists are aware that the true probability distributions of maxima discharges of rivers are not being identified [28]. Different probability distributions can be fitted to the observed time series of annual maximum discharges, and these distributions can forecast various discharge values for a chosen annual probability of exceedance. Vice versa, the same water discharge value, depending on different probability distributions, can have various annual probabilities of exceedance [43, 45-49]. For example, as is shown in Fig. 1, depending on the probability distributions, the same annual exceedance probability of 1% corresponds to different values of water discharge maxima: 2425 m³/s for the

Three-parameter Krytskyi-Menkel distribution (KM3), 2622 m³/s for the Logarithmic Pearson type III distribution (LP3), and 3633 m³/s for the Logarithmic Extreme value type I distribution (LEV1) (Gumbel type I distribution for logarithms [47]). As is seen in Fig. 1, taking into account the observed outlier (2645 m³/s) using the LEV1 distribution affected prediction results significantly. The prediction uncertainty in predicting the discharge of 1% probability of exceedance ranged from 2425 m³/s to 3633 m³/s (relative prediction error up to 50%). In terms of annual probabilities of exceedance, the prediction uncertainty ranged from 1% to 2.5% (relative prediction error up to 150%).



Probability distributions: 1 – KM3 ($C_V = 0.6, C_S = 5 C_V$); 2 – LP3; 3 – LEV1; Observed data - obtained using the Weibull plotting formula

Fig. 1. Time series (a) and probability distributions of peak discharges (b) (The Stryi River, the Verkhnye Syn’ovydne gauge station, Ukraine) [47]

Many analytical probability distributions might be used in predicting peak discharges of floods [29, 34, 35, 50-52]. In the national standards regulating hydrological calculations, different countries recommend using different parametric probability distributions. Some of the most known standardized probability distribution function types adopted for frequency analysis of peak discharges in different countries are shown in Table 1.

Table 1. Standardized probability distribution function types used in frequency analysis of peak discharges of floods in different countries [53]

Recommended probability distribution function types	Country
Pearson type III distribution (P3)	China, Switzerland
Logarithmic Pearson type III distribution (LP3)	The US, Canada, India
Generalized extreme value distribution (GEV)	Great Britain, France
Two, Three parameters log-normal distribution (LN2, LN3)	Japan
Extreme value type I distribution (Gumbell type I, EV1)	Germany, Sweden, Norway
Extreme value type I, type III distribution (EV1, EV3)	Great Britain, France
Krytskyi-Menkel three-parameter distribution (KM3)	Ukraine, former USSR' countries

In general, there is no proper theoretical or another similar justification for choosing an appropriate probability distribution to predict peak discharges of floods using observed data [49]. Therefore, any of them might be considered a permissible hypothesis. For any probability distribution, which meets the given statistical criteria and other considerations for the adequacy of simulation [47, 54], it will hardly find a sufficiently weighty reason to reject it as a possible option indisputably.

However, there is one more challenge relating to the above question. The challenge is the choice of an unbiased empirical formula to plot the observed data. The attitude that the criterion for the choice of a desirable plotting position formula may be arbitrary is rebuked easily [55]. It should be noted, to date, more than seventeen different plotting position formulas have been proposed by hydrologists and statisticians [56].

As it is known, plotting position formulas provide a non-parametric means to estimate the observed data probability distribution. Using a plotting position formula, a plot of the estimated values from a theoretical parametric probability distribution can be compared with the observed data. In particular, probability plots allow a visual examination of the adequacy of the fit provided by alternative parametric probability distributions. For example, empirical probabilities of the observed peak discharges of the Stryi River at the Verkhnye Syn'ovydneye gauge station, which are shown in Fig.1 to examine the adequacy of the fit provided by three alternative parametric probability distributions (KM3, LP3 and the Gumbel type I distribution for logarithms), were calculated using the Weibull plotting position formula:

$$P_m = \frac{m}{n+1}, \quad (1)$$

where P_m is the empirical probability of exceedance of the m -th order observed value, m is the rank of the observed value, where the highest one being "1", and n is the number of observed statistics.

Probability papers and probability plotting positions to estimate observed data probability distributions were used by hydrologists as early as 1896 [57]. The first plotting position formula used in hydrological investigations was probably one proposed by Hazen (1914) [57]:

$$P_m = \frac{m-0.5}{n}. \quad (2)$$

Lebedev (1952) and Chegodaev (1965) proposed the use of

$$P_m = \frac{m-0.3}{n+0.4}, \quad (3)$$

which is approximately the median position advocated by Johnson (1951) [57].

In turn, Blom (1958) suggested

$$P_m = \frac{m - a}{n - 2 \cdot a + 1}, \tag{4}$$

where a is a constant (usually $0 \leq a \leq 1$), which defines possible plotting positions as special cases [57].

In this study, we took into account thirteen well-known plotting position formulas. They appear the most frequently in the hydrological literature [29, 55-62]. These formulas are shown in Table 2.

Table 2. Plotting position formulas used in the study

No	Author (year)	Formula to calculate P_m	Recommended probability distributions
1	Hazen (1914)	$\frac{m - 0.5}{n}$	GEV, Gumbell type I (EV1)
2	Gringorten (1963)	$\frac{m - 0.44}{n + 0.12}$	GEV, Gumbell type I (EV1)
3	Nguyen et al. (1989)	$\frac{m - 0.42}{n + 0.3C_s + 0.05}$, C_s is skewness	P3, $-3 \leq C_s \leq 3$, and $5 \leq n \leq 100$
4	Cunnane (1978)	$\frac{m - 0.4}{n + 0.2}$	GEV, EV3, P3, LP3
5	Blom (1954)	$\frac{m - 3/8}{n + 1/4}$	LN2, LN3, LP3
6	Hosking (1990)	$\frac{m - 0.35}{n}$	Some 3-parameter distributions
7	Tukey (1962)	$\frac{m - 1/3}{n + 1/3}$	All distributions
8	Goel (1993)	$\frac{m - 0.02C_s - 0.32}{n - 0.04C_s + 0.36}$	GEV
9	Beard (1945)	$\frac{m - 0.3175}{n + 0.365}$	All distributions
10	Kim et al. (2012)	$\frac{m - 0.32}{n + 0.0149C_s^2 - 0.1364C_s + 0.3225}$	GEV
11	Lebedev (1952), Chegodaev (1965)	$\frac{m - 0.3}{n + 0.4}$	GEV, EV3, P3, LP3, KM3
12	Adamowski (1985)	$\frac{m - 0.25}{n + 0.5}$	EV1, GEV, EV3
13	Weibull (1939)	$\frac{m}{n + 1}$	All distributions

Table 2 also shows which plotting position formula can be the best to fit different parametric probability distributions. It should be noted that the choice of the best plotting position formula for fit to the different probability distributions has been discussed many times in hydrology and statistical literature [55-62]. However, a more worthwhile criterion for choosing plotting position formulas might be based on obtained empirical estimates of plotting position probabilities. It might be better

than a comparison of empirical plotting position probabilities with the theoretical probabilities to test individual probability distributions. Eventually, we will know which theoretical probability distribution is better in a contest of obtained estimates of future events only after these events happen. However, in terms of decision-making, we may consider all recommended plotting position formulas as admissible options to test each of recommended theoretical probability distributions and choose the best one.

This paper proposes a new numerically-analytical method to predict the design maxima discharges of floods using empirical estimates of plotting position probabilities obtained by different plotting position formulas. The proposed method is tested on a fragment of a time series of the maximum discharges of the Uzh River, the Tisza River basin, Transcarpathia region, observed at the hydrological station (HS) “Uzhhorod”.

3. Case study and objectives of this paper

The Uzh River belongs to the Tisza river basin, originates in the mountains in the northwest of the Transcarpathia region of Ukraine, and flows into the Laborec River in eastern Slovakia. The major part of the Uzh basin is in Ukraine (Fig. 2). The river length is 132.4 km, and its catchment area is 2,790.9 km² [63] (in Ukraine 112.8 km and 1,970 km²) [64, 65]. In the upper reaches, Uzh has a pronounced mountainous character (slopes of the river channels 5-20 m/km). Its lower parts belong to the lowland (bed slopes of 2-0.3 m/km or less [63]).

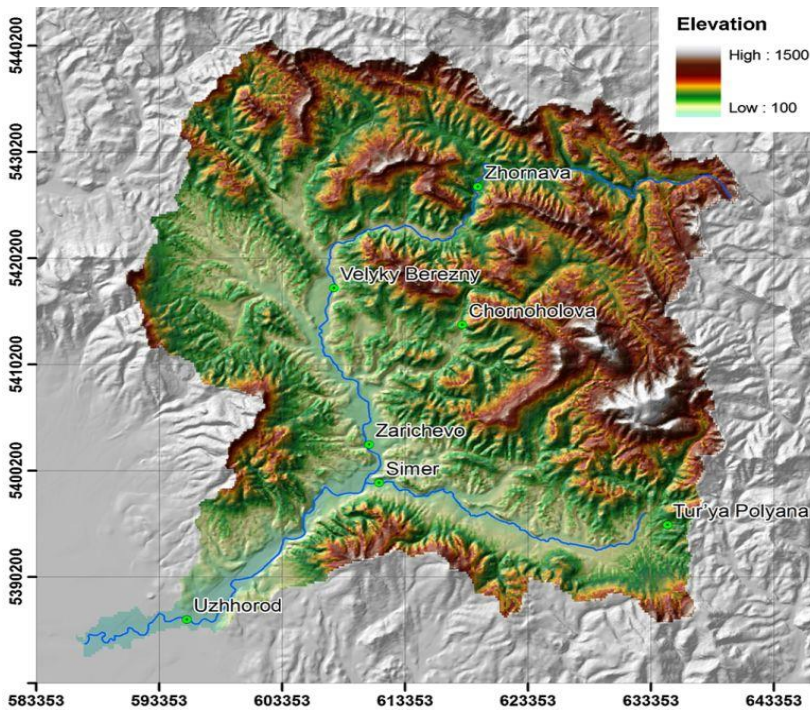


Fig. 2. The Uzh River watershed and catchment topography; coordinates are in UTM, 34 N zone (taken from [64])

In the lowlands, Uzh flows within the city of Uzhhorod – the administrative centre and the largest town (125,000 inhabitants) of the Transcarpathia region. The river width is mainly 15-30 m; near Uzhhorod, it reaches up to 135 m. The river valley width varies from 15 m in the upper reaches to 100-300 m in the downstream, and in the lowlands, it reaches 2-2.5 km. The river banks are steep, 1-2 m high, sometimes up to 6-8 m, the river bottom in the upper and middle reaches is rocky, and in Uzhhorod and downstream, it is silted up [65].

The runoff of the Uzh River is very variable. It is only about 29.6 m³/s of the mean annual water discharge near Uzhhorod [65, 66], a minimum 7-day summer-autumn flow can decrease below 2 m³/s [67], but a maximum peak one can exceed 1,000 m³/s and more during floods. The Uzh River is known for its heavy snowmelt and rain flash floods, which can occur 3-8 times per year. Admittedly, catastrophic floods are an inherent element of the hydrological regime of rivers in the Transcarpathia [68] and the Uzh River basin is one of the most flood-prone river basins of Ukraine [64]. Floods in the Uzh basin were recorded in all seasons of the year and can be showery, snowy and snow-flurry by origin; the most significant floods are formed in the cold seasons (late autumn, winter, and early spring) but they occur also in the summer season, the phenomenon is being influenced by the moisture intake brought by the air masses [25]. Among months, the richest ones for water are January, March and November [25].

Six hydrometeorological gauging stations are in the Uzh basin on Ukrainian territory. Measurements have taken place for more than 10 years [64]. These gauging stations belong to Joint Ukrainian-Hungarian Automated Information-Measuring System for flood forecasting and management in the Tisza River basin in the Transcarpathian region (AIMS TISZA) [69, 70]. They measure precipitation, temperature, water levels, and discharges [64]. Additionally, the Zhornava station measured flood discharges from 1952, the Zarichevo station – from 1947, and the Uzhhorod hydrological station (HS “Uzhhorod”) – from 1947.

This case study relates to peak water discharges measured at the HS “Uzhhorod”. **The objectives of this paper are:** (1) to develop a numerically-analytical method for prediction of design maxima discharges of floods using empirical estimates of plotting positions; (2) to test the method when making predictions of maxima discharges of 0.5%, and 1% annual exceedance probabilities using a fragment of a time series of the observed data for the Uzh River, the hydrological station (HS) “Uzhhorod”.

4. Materials and methods

A time series of maximum discharges of the Uzh River, which were observed at the hydrological station (HS) “Uzhhorod” from 1947 to 1999 (Fig. 3), was employed in this study. The data were taken from the Hydrological Yearbooks of the Central Geophysical Observatory named after B. Sreznevsky [71].

The data sample length is 53 years. The maximum observed peak discharge value within the data sample is 1680 m³/s (1957); the minimum value is 146 m³/s (in 1961). The mean peak discharge within the data sample is 689 m³/s; the sample standard deviation – of 364 m³/s. The coefficient of variation of the time series C_V is 0.53, the skewness C_S – of 0.52, and the C_S / C_V is 0.99.

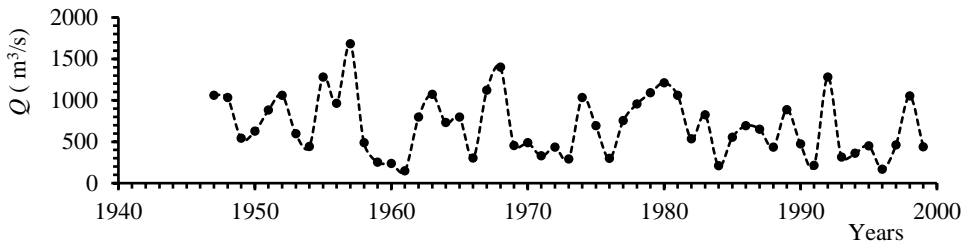


Fig. 3. Time series of annual maximum water discharges, the Uzh River, HS “Uzhgorod”, the data sample of 1947-1999

In the study, the following methods were used: (1) generalised scientific methods of theoretical and empirical research, analysis and synthesis, expert evaluation and comparison, formalization and modelling, and extrapolation methods [45, 72]; (2) fundamental methods of probability theory and mathematical statistics, and risk theory [28, 55, 57], in particular, regarding risk assessment and management [27, 31, 32]; (3) specific statistical methods in hydrology [29, 34, 35, 50-52, 59]; (4) utility theory methods [73, 74] and decision making methods under risk and uncertainty [43, 45, 75, 76].

Thirteen plotting position formulas were used in the study. These are shown in Table 2. The plotting position formulas were considered in terms of possible expert suggestions for assessing the annual empirical probabilities of exceedance of observed maxima discharges. As possible theoretical alternatives for predicting design maxima discharges of the Uzh River at the HS “Uzhhorod” considered were five probability distributions (Fig. 4): 1) the Kritskyi-Menkel three-parameter distribution (KM3) ($C_V = 0.53$, $C_S = C_V$); 2) Pearson’s type III distribution (P3) ($C_S = 0.52$); 3) the Extreme value type I distribution (Gumbell’s type I distribution, EV1); 4) the Logarithmic Pearson type III distribution (LP3) ($C_S = -0.44$); and 5) the Two parameters logarithmic-normal distribution (LN2).

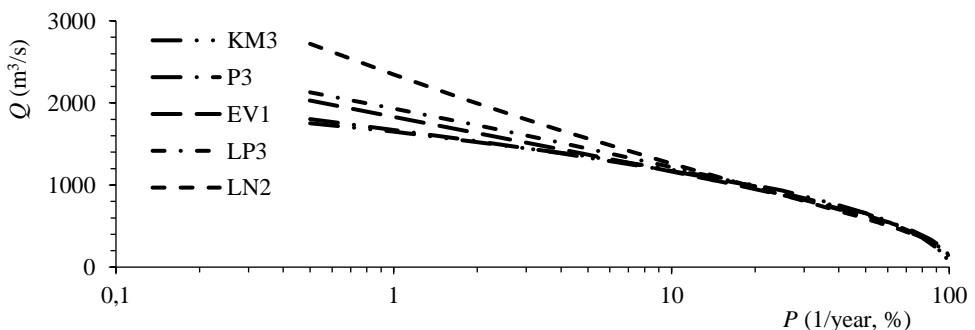


Fig. 4. Alternative parametric probability distributions of annual maxima discharges of the Uzh River at the HS “Uzhhorod” for the data sample of 1947-1999

The population parameters of the parametric probability distributions (Fig. 4) were estimated from the sample statistics by the method of moments; the sample characteristics were equated to the population parameters.

5. Data pre-analysis and several preliminary summarising remarks

The data pre-analysis included calculating empirical annual probabilities of exceedance P_m observed maxima discharge employing various plotting position formulas depending on the rank $m = 1, \dots, n$ of the observed value, where the highest one has the rank $m = 1$, and $n = 53$ is the number of observed data. Fig. 5 shows the results of the calculations using the six most-cited formulas (Hazen, Gringorten, Blom, Tukey, Chegodaev, and Weibull). Fig. 6 shows the Hazen, Chegodaev, and Weibull plot positions in comparison with the chosen alternative parametric probability distributions. Below, Table 3 shows the results of the calculations of empirical probabilities of exceedance for the six maxima discharges ($m = 1, 2, 3, 5, 6$, and 12) employing all taken into account (thirteen, See Table 2) plot position formulas.

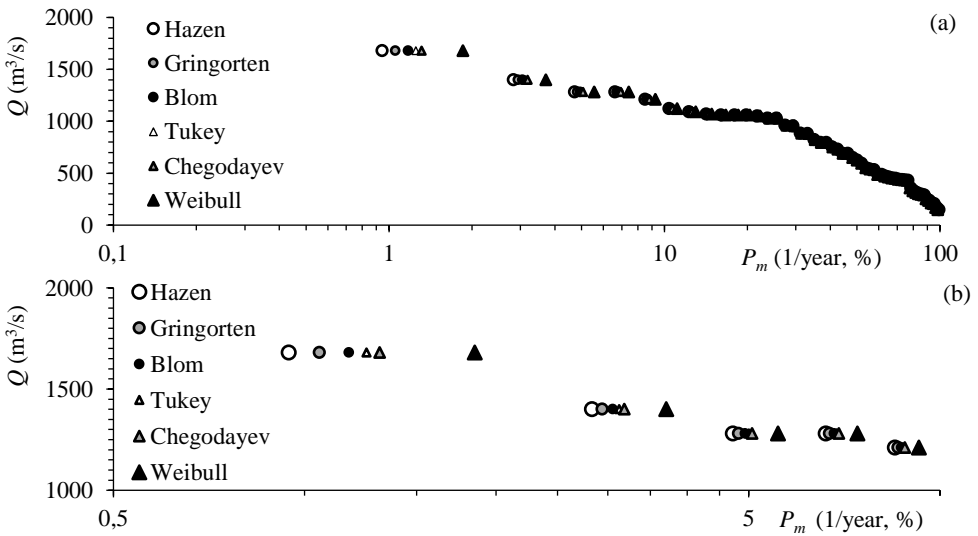


Fig. 5. Empirical probabilities of exceedance P_m according to plotting positions: (a) within 0.1-100%; (b) within 0.5-10%; annual maxima discharges of the Uzh River, the HS “Uzhhorod”, the data sample of 1947-1999

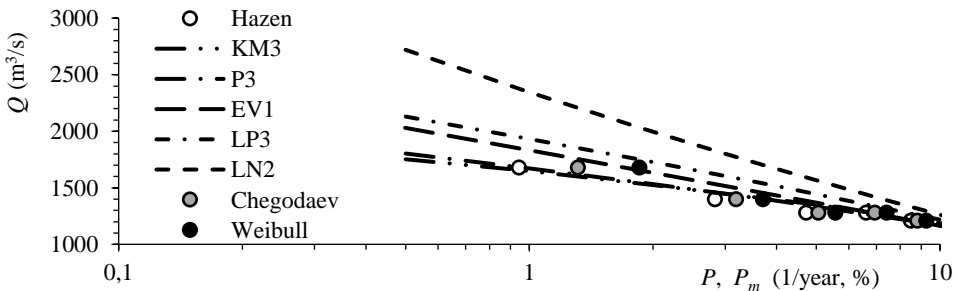


Fig. 6. Hazen's, Chegodaev's, and Weibull's plot positions in comparison with the chosen alternative parametric probability distributions

Table 3. Empirical probabilities of exceedance P_m for the observed maxima discharges of 1680, 1400, 1280, 1210, 1120, and 1050 m³/s ($m = 1, 2, 3, 5, 6,$ and 12) depending on the different plotting position formulas

No	Plotting position formula (author)	P_m (1/year, %)					
		$m = 1$	$m = 2$	$m = 3$	$m = 5$	$m = 6$	$m = 12$
		1680	1400	1280	1210	1120	1050
1	Hazen	0.94	2.83	4.72	8.49	10.38	21.70
2	Gringorten	1.05	2.94	4.82	8.58	10.47	21.76
3	Nguyen et al.	1.09	2.97	4.85	8.61	10.49	21.76
4	Cunnane	1.13	3.01	4.89	8.65	10.53	21.80
5	Blom	1.17	3.05	4.93	8.69	10.56	21.83
6	Hosking	1.23	3.11	5.00	8.77	10.66	21.98
7	Tukey	1.25	3.13	5.00	8.75	10.63	21.88
8	Goel	1.26	3.13	5.00	8.75	10.63	21.88
9	Beard	1.28	3.15	5.03	8.77	10.65	21.89
10	Kim et al.	1.28	3.15	5.03	8.79	10.67	21.93
11	Chegodaev	1.31	3.18	5.06	8.80	10.67	21.91
12	Adamowski	1.40	3.27	5.14	8.88	10.75	21.96
13	Weibull	1.85	3.70	5.56	9.26	11.11	22.22

It can be easily noted that increasing the predicting horizon toward low probable and more extreme events increases the difference (or divergence) between the estimates of probabilities obtained using the different parametric probability distributions and the different plotting position formulas.

For estimates calculated employing the different plotting position formulas (Table 3), we tried to quantify this difference using the parameter, which was named the divergence indicator d_m :

$$d_m = \frac{P_m(W)}{P_m(H)}, \text{ or } d_m = \frac{T_{r,m}(H)}{T_{r,m}(W)}, \tag{5}$$

where $m = 1, 2, 3, 5, 6,$ and 12 is the rank of the observed maxima discharges of 1680, 1400, 1280, 1210, 1120, and 1050 m³/s; $P_m(W)$, $P_m(H)$ are the empirical probabilities of exceedance, and $T_{r,m}(W)$, $T_{r,m}(H)$ are the return periods of the observed maxima discharges calculated using the Weibull (1) and Hazen (2) plotting position formulas, correspondingly.

The results of the divergence indicator calculations are shown in Table 4 and Fig. 7. In particular, Fig. 7 shows two dependencies relating to the indicator d_m : (a) between the discharges' return periods calculated using the Weibull and Hazen formulas and the divergence indicator; (b) between the indicator d_m and the observed peak discharges.

Table 4. Divergence indicator d_m values for empirical probabilities obtained by the Weibull and Hazen plotting position formulas

Plotting position formula (author)	Q_m (m ³ /s); P_m (1/year, %) \ $T_{r,m} = 100 \cdot P_m^{-1}$ (years)					
	$m = 1$	$m = 2$	$m = 3$	$m = 5$	$m = 6$	$m = 12$
	1680	1400	1280	1210	1120	1050
Weibull	1.85 \ 54	3.70 \ 27	5.56 \ 18	9.26 \ 11	11.11 \ 9	22.22 \ 5
Hazen	0.94 \ 106	2.83 \ 35	4.72 \ 21	8.49 \ 12	10.38 \ 10	21.70 \ 5
d_m	1.96	1.31	1.18	1.09	1.07	1.02

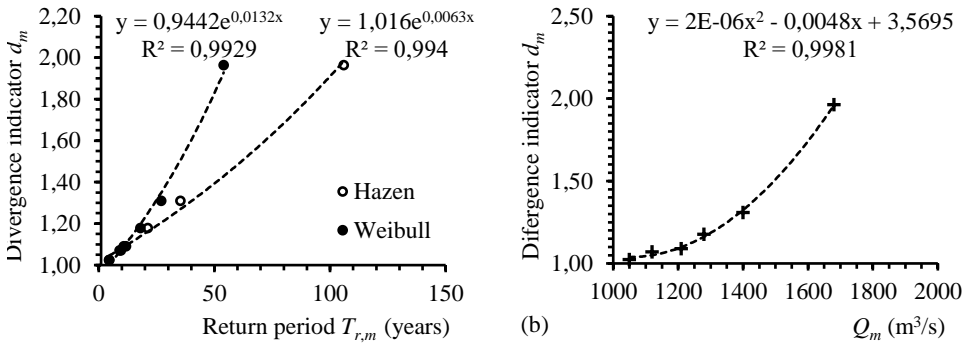


Fig. 7. Dependencies: (a) between the discharge return period $T_{r,m}$ and the divergence indicator d_m ; (b) between the observed peak discharge Q_m and the indicator d_m

Summarising, several preliminary remarks can be made.

The first remark concerns the events with short return periods. The different plotting position formulas provide nearly similar results. These events have return periods of 5 years or less in this case study. The annual probabilities of exceedance are 20% and more. The same conclusion applies to the chosen alternative parametric probability distributions (Fig. 4).

The second remark relates to the difference in the empirical estimates of the probability of exceedance provided by different plotting position formulas. This difference increases as the frequency of the occurred events decreases (See Fig. 5, 6, and Table 3). The same conclusion applies to the probabilities of exceedance for future events, the estimates that the alternative parametric probability distributions predict (Fig. 4).

The third remark concerns choosing a better parametric probability distribution among possible alternatives. Plot position formulas can influence the decision and, accordingly, the prediction of the design peak discharge value of a given small probability of exceedance. For example, visually (See Fig. 6), the Hazen plotting position formula compels us to pay attention to the Kritskyi-Menkel three-parameter generalised gamma distribution (KM3) and Pearson's type III distribution (P3); the Chegodaev formula indicates the Extreme value type I distribution (Gumbell's type I distribution, EV1); eventually, the Weibull formula does not exclude employing the Logarithmic Pearson type III distribution (LP3).

The regressions $d_m = f(T_{r,m})$ (See Fig. 7a) indicate that further enlarging of the return period of the observed peak discharge may correspond to an increase in the divergence in plotting position estimates the different formulas provide. This divergence depends on the plotting position formulas chosen to be compared. The regression $d_m = f(Q_m)$ (See Fig. 7b) indicates that further enlarging of the observed peak discharge may also correspond to an increase in the divergence in plotting position estimates the different formulas provide. By estimating the divergence indicator and building these regressions, we can provide predictions based on extrapolation. In the first step, the prediction is implemented using the direct dependencies between the divergence indicator values and the design discharge return period values. In the second step, it is used the dependence between the discharges and the divergence indicator values. Predicting design discharges is made using an iterative calculation method.

6. Developing and testing the proposed method

6.1. Technique to apply the method

Fig. 8, and Table 5 show results of predicting the design maxima discharge of 1% and 0.5% annual probability of exceedance for the Uzh River, the HS “Uzhhorod”. These predictions were based on empirical probabilities obtained by the Weibull and Hazen plotting position formulas.

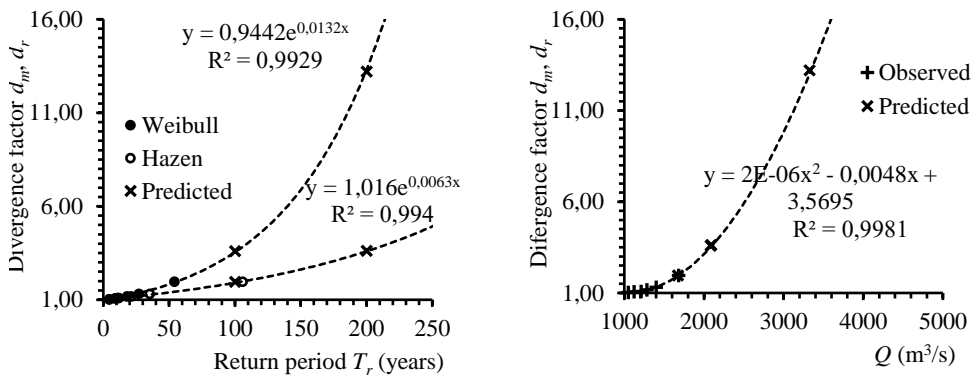


Fig. 8. Predicting the design maxima discharges of 1% and 0.5% annual probabilities of exceedance for the Uzh River, the HS “Uzhhorod”

Table 5. Results of predicting the design maxima discharges of 1% and 0.5% annual probabilities of exceedance for the Uzh River, the HS “Uzhhorod”

P (1/year, %)	$T_r = 100 \cdot P^{-1}$ (years)	Design maxima discharge Q (m ³ /s) according to:	
		Hazen	Weibull
1	100	1670	2080
0.5	200	2090	3323

Fig. 9 shows the predicted values of the design maxima discharges of the 1% and 0.5% annual probabilities of exceedance. The prediction results are compared with the chosen alternative parametric probability distributions.

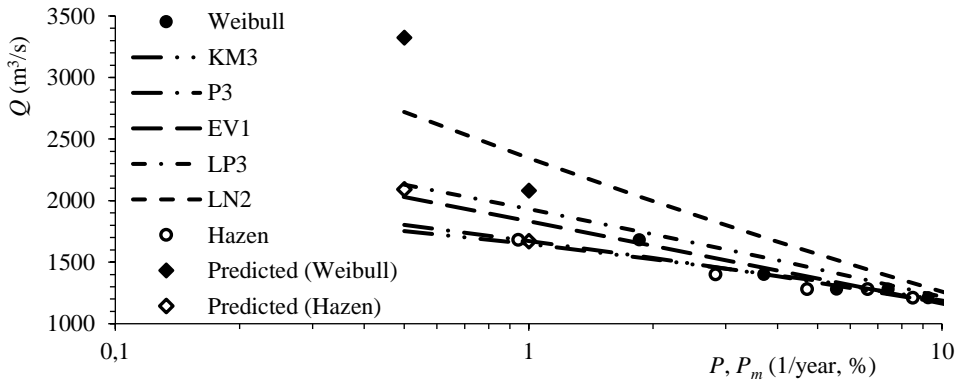


Fig. 9. Predicted values of the design maxima discharges of the 1% and 0.5% annual probabilities of exceedance in comparison with the chosen alternative parametric probability distributions (the Uzh River, the HS “Uzhhorod”)

It is suggested the following technique to apply the proposed numerically-analytical method for prediction design maxima discharges of floods using empirical estimates of plotting positions.

Stage 1: Preparation of a time series of maximum water discharges;

Stage 2: Reviewing different plotting position formulas;

Stage 3: Calculating empirical annual probabilities of exceedance P_m (1/year, %) of observed maxima discharges employing chosen plotting position formulas depending on the rank $m = 1, \dots, n$ of the observed discharge values, where the highest one has the rank $m = 1$, and n is the number of observed discharges;

Stage 4: Choosing a plotting position formula (formulas) for prediction design maxima discharges;

Stage 5: Choosing a counterparty plotting position formula (formulas) to calculate the divergence indicator d_m values; the counterparty plotting position formula may be chosen as an arbitrary one; it may be one of the formulas providing marginal (maximum, minimum) plotting positions (for example, Hazen’s or Weibull’s formulas);

Stage 6: Computing the divergence indicator d_m values;

Stage 7: Choosing a population of plot positions ($m = 1, \dots$) for which $d_m > 1$ and building the regression $d_m = f(T_{r,m})$; predicting the divergence indicator $d_r = f(T_r)$ for the chosen design return period T_r of the design maxima discharge by using the extrapolation method;

Stage 8: Building the regression $d_m = f(Q_m)$; predicting the design maxima discharge for the chosen design return period $T_r = 100 \cdot P^{-1}$, where P is a chosen design annual probability of exceedance (1/year, %), by using the extrapolation and iterative calculation methods.

6.2. Using the Fishburne rule

Results obtained by using different plotting position formulas may be considered expert estimates. These expert estimates may be given different importance in making decisions under uncertainty and risk [73-76]. For example, in flood management strategies, the plotting position estimates obtained using the Weibull formula contribute to choosing more cautious decision options. However, more cautious solutions may be associated with higher capital costs. In turn, the plotting position estimates obtained by Hazen's formula contribute to choosing decision options with lower capital costs. However, these less costly decision options may inflict increasing in future flood losses.

Accordingly, making decisions, different plotting position formulas can be considered indicators of the predisposition to more cautious or less expensive decision options. In other words, different plotting position estimates obtained using different plotting position formulas can acquire their weight level in a system of indicators' importance under the decision-making process.

An optimal distribution of the weights of the indicators from the point of view of informational entropy is referred to as Fishburne's rule. The Fishburne rule considers that the level of indicators' importance is determined only by arranged in descending order of importance.

According to the Fishburne rule, the "weight" w_i for the i -th plotting position estimate $P_{m,i}$ obtained using the i -th formula can be calculated as [73, 74]:

$$w_i = \frac{2(k-i+1)}{(k+1) \cdot k}, \quad (6)$$

where i is the rank of the i -th plotting position estimate obtained using the i -th formula taking into account the level of the formula importance; the highest estimate gets the rank $i = 1$ when there is a predisposition to more cautious options, and vice-versa, when there is a predisposition to options with lower capital costs, the smallest one has the rank $i = k$; k is the total number of the ranked-set plotting position estimates (formulas).

Then, the rank-weighted estimate of the annual plotting position probability $P_{m,w}$ depending on the selected significance option of the different plotting position formulas

$$P_{m,w} = \sum_{i=1}^k P_{m,i} \cdot w_i, \quad (7)$$

where m is the rank of the observed peak water discharge Q_m (m^3/s).

Using the Fishburne rule enables getting two possible rank-weighted estimates of the annual plotting position probability $P_{m,w}$ depending on the selected significance option of the different plotting position formulas: the rank-weighted upper bound estimate (sup) $P_{m,w}^{\text{sup}}$, the rank-weighted lower bound estimate (inf) $P_{m,w}^{\text{inf}}$. The rank-weighted upper bound estimate $P_{m,w}^{\text{sup}}$ corresponds to the predisposition to more

cautious decision options. The rank-weighted lower bound estimate $P_{m,w}^{inf}$ corresponds to the predisposition to less expensive decision options.

For $k = 13$, the following weights of the i -th different plotting position estimates (formulas) were obtained depending on their rank: ($i = 1, w_1 = 0.143$); (2, 0.132); (3, 0.121); (4, 0.110); (5, 0.099); (6, 0.088); (7, 0.077); (8, 0.066); (9, 0.055); (10, 0.044); (11, 0.033); (12, 0.022); ($i = 13, w_{13} = 0.011$).

Table 6 shows the calculated parameters, where the divergence indicator $d_m = P_{m,w}^{sup} / P_{m,w}^{inf}$, the return periods: $T_{r,m}^{sup} = 100 / P_{m,w}^{sup}$, $T_{r,m}^{inf} = 100 / P_{m,w}^{inf}$.

Table 6. The rank-weighted upper bound $P_{m,w}^{sup}$ and lower bound $P_{m,w}^{inf}$ estimates for empirical probabilities of observed maxima discharges

Parameters	Observed maxima discharge Q (m ³ /s)					
	1680	1400	1280	1210	1120	1050
m	1	2	3	5	6	12
$P_{m,w}^{sup}$ (%)	1.35	3.22	5.09	8.84	10.71	21.95
$P_{m,w}^{inf}$ (%)	1.15	3.03	4.91	8.67	10.55	21.82
d_m	1.168	1.061	1.036	1.019	1.015	1.006
$T_{r,m}^{sup}$ (years)	74	31	20	11	9	5
$T_{r,m}^{inf}$ (years)	87	33	20	12	9	5

Below, Fig. 10 and Table 7 show the predicted values of the design maxima discharges of 1% and 0.5% annual probabilities of exceedance for the Uzh River, the HS “Uzhhorod”. They were obtained according to the data in Table 6. Fig. 11 compares the obtained prediction results with the chosen alternative probability distributions.

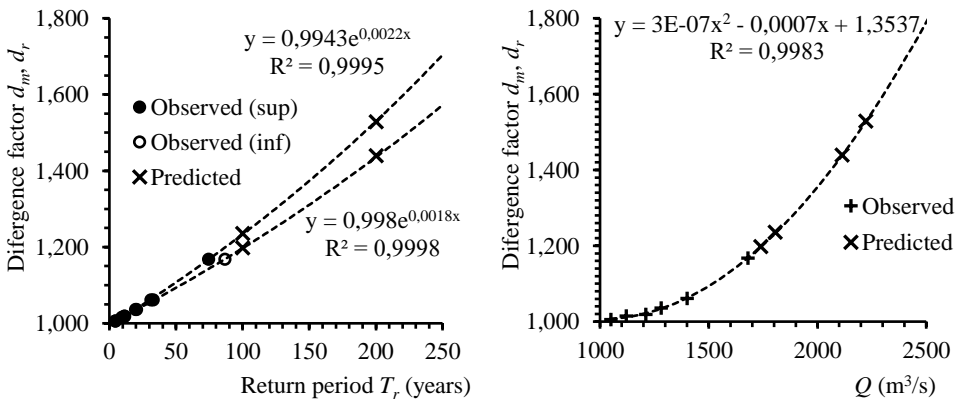


Fig. 10. Predicting the design maxima discharges of 1% and 0.5% annual probabilities of exceedance for the Uzh River, the HS “Uzhhorod”, according to data in Table 6

Table 7. Results of predicting the design maxima discharges of 1% and 0.5% annual probabilities of exceedance for the Uzh River, the HS “Uzhhorod”, using the Fishburne rule

P (1/year, %)	$T_r = 100 \cdot P^{-1}$ (years)	Design maxima discharge Q (m ³ /s) obtained using the Fishburne rule	
		inf	sup
1	100	1738	1805
0.5	200	2113	2222

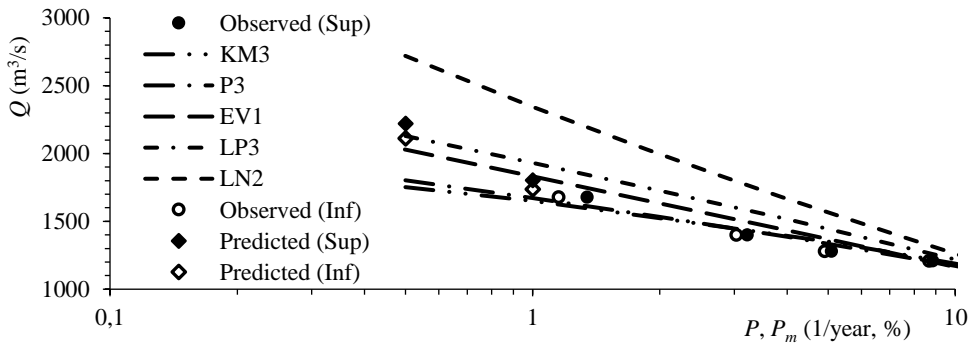


Fig. 11. Comparison of the obtained prediction results (Table 7) with the chosen alternative parametric probability distributions

It is worth noting the goodness of fit of the peak discharges of 1% probability of exceedance obtained by extrapolation of plotting position probabilities using the proposed method to the Extreme value type I distribution (Gumbell type I, EV1). The design discharge of 1% probability of exceedance obtained using the EV1 distribution is 1832 m³/s. The upper bound estimate (sup) of such a discharge using the proposed method and the Fishborn rule is 1805 m³/s. The relative prediction error is less than 1.5%. The lower bound estimate (inf) of such a discharge using the proposed method and the Fishborn rule is 1738 m³/s. The relative prediction error is 5.4%. However, it is worth noting the goodness of fit of the peak discharges of 0.5% probability of exceedance obtained by extrapolation of plotting position probabilities using the proposed method to the Logarithmic Pearson type III distribution (LP3). The design discharge of 0.5% probability of exceedance obtained using the LP3 distribution is 2130 m³/s. The upper bound estimate (sup) of such a discharge using the proposed method and the Fishborn rule is 2222 m³/s. The relative prediction error is approximately 4.2%. The lower bound estimate (inf) of such a discharge using the proposed method and the Fishborn rule is 2113 m³/s. The relative prediction error is less than 0.8%.

7. Some discussion remarks

Is epistemic or non-stochastic uncertainty a challenge in predicting extreme hydrological characteristics? Yes, it is. It can be a challenge. However, at least, the multi-model approach may promote revealing epistemic uncertainty.

To answer this question, an original method of prediction was developed. The method was called a numerically-analytical method. It is based on using different plotting position formulas, numerical calculations of plotting position probabilities, and extrapolation of the divergence between the obtained estimates.

This method may support to choice of a better parametric probability distribution. There is no proper theoretical or another similar justification for choosing an appropriate probability distribution to predict peak discharges of floods using observed data. This method may promote such a justification.

The estimates predicted by this method are also noteworthy. In terms of predicting accuracy, these estimates are no different principally from estimates that can be obtained using parametric probability distributions.

Conclusions

1. Plotting position formulas provide a non-parametric means to estimate the observed data probability distribution. Using a plotting position formula, a plot of the estimated values from a theoretical parametric probability distribution can be compared with the observed data. It allows a visual examination of the adequacy of the fit provided by alternative parametric probability distributions.

2. There are more than seventeen different plotting position formulas to fit theoretical parametric probability distributions with the observed data. The issue is the choice of an unbiased empirical formula to plot the observed data. Any plotting position formula can be an option for fitting parametric probability distributions.

3. Results of calculating empirical annual probabilities of exceedance observed maxima discharge employing various plotting position formulas show that increasing the predicting horizon toward low probable, more extreme events increases the divergence between the estimates obtained using the different plotting position formulas. It is reasonable to assume that this divergence may be extrapolated.

4. An original numerically-analytical method is developed to predict design maxima discharges of floods using empirical estimates of plotting positions. It is based on using different plotting position formulas, numerical calculations of plotting position probabilities, and extrapolation of the divergence between the obtained estimates. The method is tested in predicting the maxima discharges of 0.5% and 1% annual probability of exceedance for the Uzh River, the hydrological station (HS) "Uzhhorod".

5. Among the practically significant results of the study, the following ones should be highlighted. The upper bound estimate (sup) of the design peak discharge of 1% probability of exceedance obtained by extrapolation of plotting position probabilities using the proposed method and the Fishborn rule is 1805 m³/s. It fits the estimate of 1832 m³/s derived from the Extreme value type I distribution (Gumbell type I, EV1). The upper bound estimate (sup) of the design peak discharge of 0.5% probability of exceedance using the proposed method and the Fishborn rule is 2222 m³/s. It fits the estimate of 2130 m³/s derived from the Logarithmic Pearson type III distribution (LP3). Thus, depending on different design probabilities of exceedance, the proposed method may support the choice of a better parametric probability distribution to predict peak discharges.

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АПРОБАЦІЯ ЧИСЕЛЬНО-АНАЛІТИЧНОГО МЕТОДУ ПРОГНОЗУВАННЯ РОЗРАХУНКОВИХ МАКСИМАЛЬНИХ ВИТРАТ ПАВОДКІВ ІЗ ВИКОРИСТАННЯМ ФОРМУЛ ЕМПІРИЧНОЇ ЙМОВІРНОСТІ: ПРИКЛАД РІЧКИ УЖ, ДАНІ ГІДРОЛОГІЧНОЇ СТАНЦІЇ «УЖГОРОД»

Анотація. Існує багато аналітичних розподілів ймовірності, які можна використовувати для прогнозування пікових витрат повеней. Однак, належного теоретичного чи іншого подібного обґрунтування для вибору відповідного параметричного розподілу ймовірності для прогнозування пікових витрат повеней на основі даних спостережень не існує. Будь-який із рекомендованих розподілів ймовірності можна вважати допустимою гіпотезою, якщо він відповідає заданим статистичним критеріям і при цьому враховуються інші міркування щодо адекватності

моделювання. У свою чергу, для порівняння теоретичних параметричних розподілів ймовірності із спостережуваними даними було запропоновано понад сімнадцять різних формул для розрахунку емпіричних ймовірностей подій, що відбулися. І, з точки зору прийняття рішень, усі ці формули також слід розглядати як допустимі варіанти при підгонці теоретичних розподілів ймовірності та виборі серед них кращого в якості моделі.

Незважаючи на те, що вибір кращої формули емпіричної ймовірності для підгонки й порівняння різних розподілів ймовірностей багато разів обговорювався в гідрологічній та статистичній літературі, досі не запропоновано жодного конкретного критерію для вибору серед цих формул. Можливо, такий критерій взагалі і не потрібен. Можливо, більше значення має різноманітність оцінок, які можна отримати за допомогою цих формул. Як відомо, формули емпіричної ймовірності забезпечують достатньо прості непараметричні засоби для оцінки розподілу ймовірностей спостережуваних даних, тому з точки зору інформаційної ентропії ці різні оцінки дозволяють виявити епістемічну (нестохастичну або суб'єктивну) невизначеність у прогнозах гідрологічних екстремумів.

Результати розрахунку емпіричних річних ймовірностей перевищення спостережуваних максимальних витрат води паводків із застосуванням різних формул емпіричної ймовірності показують, що збільшення горизонту прогнозування в бік малоймовірних, більш екстремальних подій збільшує розбіжність між оцінками, отриманими за допомогою різних формул, що при цьому використовуються. Таким чином, розумно припустити, що ця розбіжність може бути екстрапольована для прогнозу розрахункової максимальної витрати на основі отриманих емпіричних оцінок ймовірності витрат, що спостерігалися.

У цій статті пропонується оригінальний чисельно-аналітичний метод із використанням такої екстраполяції. Він заснований на використанні різних формул емпіричної ймовірності, чисельних розрахунках емпіричних ймовірностей та екстраполяції розбіжності між отриманими оцінками. Метод апробовано при прогнозуванні максимальних витрат 0,5% та 1% річної ймовірності перевищення для річки Уж, що протікає в Закарпатській області, за даними спостережень на гідрологічній станції «Ужгород».

Ключові слова: річна ймовірність перевищення; показник розбіжності; екстраполяція; паводки; чисельно-аналітичний метод; формули емпіричної ймовірності; розподіли ймовірності; прогноз; період повторення.

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